

Mobility versus quality in 2D semiconductor structures

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We consider theoretically effects of random charged impurity disorder on the *quality* of high-mobility two dimensional (2D) semiconductor structures, explicitly demonstrating that the sample mobility is not necessarily a reliable or universal indicator of the sample quality in high-mobility modulation-doped 2D GaAs structures because, depending on the specific system property of interest, mobility and quality may be controlled by different aspects of the underlying disorder distribution, particularly since these systems are dominated by long-range Coulomb disorder from both near and far random quenched charged impurities. We show that in the presence of both channel and remote charged impurity scattering, which is a generic situation in modulation-doped high-mobility 2D carrier systems, it is quite possible for higher (lower) mobility structures to have lower (higher) quality as measured by the disorder-induced single-particle level broadening. In particular, we establish that there is no reason to expect a unique relationship between mobility and quality in 2D semiconductor structures as both are independent functionals of the disorder distribution, and are therefore, in principle, independent of each other. Using a simple, but reasonably realistic, “2-impurity” minimal model of the disorder distribution, we provide concrete examples of situations where higher (lower) mobilities correspond to lower (higher) sample qualities. We discuss experimental implications of our theoretical results and comment on possible strategies for future improvement of 2D sample quality.

I. INTRODUCTION

One of the most significant materials developments in the fundamental quantum condensed matter physics, which is not universally known outside the 2D community, has been the astonishing 3,000-fold increase in the low temperature electron mobility of GaAs-based 2D confined quantum systems from $\sim 10^4$ cm²/Vs in the first modulation-doped GaAs-AlGaAs 2D heterostructures¹ in 1978 to the current world-record mobility of $\sim 3 - 4 \times 10^7$ cm²/Vs in the best available modulation doped GaAs-AlGaAs quantum wells of today.² This represents a truly remarkable more than three orders of magnitude enhancement in the low temperature (~ 1 K) electron mean free path from a rather short microscopic length ~ 50 nm in 1978 to the essentially macroscopic length scale of ~ 0.2 mm in 2010. This incredible 3,000-fold enhancement of the 2D carrier mean free path, although much less well-known than the celebrated Moore’s law in the Si electronics industry, is actually quantitatively on par with Moore’s law increase in the microprocessor performance. Unlike Moore’s law in Si microelectronics performance, where the motivation has been technological, however, the drive for the mobility enhancement in 2D GaAs structures has been motivated entirely by fundamental physics considerations. Indeed, this increase in the 2D mobility has been accompanied by some of the most spectacular experimental discoveries in modern physics including, for example, the fractional quantum Hall effect (FQHE)³, the even-denominator FQHE⁴, the bilayer half-filled FQHE⁵, the anisotropic stripe and bubble phases⁶, and many other well-known novel 2D phenomena. (As an aside we point out that, qualitatively

similar to the situation in the Moore’s law, the exponential enhancement in the 2D mobility of semiconductor heterostructures has slowed down considerably in the recent years with only a 30% increase in the mobility during the 2003 – 2013 period, from $\sim 3 \times 10^7$ to $\sim 4 \times 10^7$ cm²/Vs, after roughly a factor of 1,000 increases during 1978 – 2003.)

We mention right in the beginning that our interest here is obviously the $T = 0$ transport properties (or low-temperature transport properties) with the temperature being much smaller than both the Fermi temperature and the Bloch-Grüneisen (BG) temperature of the 2D system so that all thermal effects have saturated, and we do not need to account for either phonon scattering or finite temperature effects in the Fermi distribution function. The high mobility of 2D semiconductor structures applies only to this low-temperature situation, and at higher temperatures (> 10 K), the mobility is dominated by phonon scattering, a situation already well-studied in the literature⁷. Our theory is just restricted only to $T = 0$ impurity-scattering-limited 2D transport properties, which limit the ultimate achievable mobility in these systems. It is also important to emphasize that we ignore all weak localization aspects of 2D transport properties, restricting entirely to the semiclassical transport behavior where the concept of a mobility is valid. Thus, the theory applies only at densities where weak-localization behavior does not manifest itself at the experimental temperatures (~ 25 mK – 2 K). At a fixed density, this limits our theory to a temperature high enough (> 1 mK) so that the phase breaking length is shorter than the semiclassical mean free path. The physics of mobility/quality dichotomy discussed in this paper satisfies all

of these constraints (i.e. no phonon scattering, no weak localization, temperature much lower than the fermi temperature) very well.

Although the *mobility* enhancement of 2D systems has generally led to the improvement of sample *quality* on the average over the years as manifested in the observation of new phenomena, it has been known from the early days that the connection between *mobility* and *quality* is at best a statistically averaged statement over many samples and is not unique, i.e., a sample with higher mobility than another sample may not necessarily have a higher quality with respect to some specific property (e.g., the existence or not of a particularly fragile fractional quantum Hall plateau). Thus, higher (lower) mobility does not necessarily always translate into higher (lower) quality for specific electronic properties. Since 2D carrier mobility is a function of carrier density⁸, in fact, it is, in principle, possible for a sample to have a higher (lower) mobility than another sample at higher (lower) carrier density, implying that the measured mobility at some fixed high density is not (always) even a good indicator of transport quality itself as a function of carrier density, let alone being an indicator for the quality of other electronic properties!

The reason for the above-mentioned mobility/quality dichotomy is rather obvious to state, but not easy to quantify. Both mobility and quality (e.g., the measured activation gap for a specific FQHE state or some other specified electronic property) depend on the full disorder distribution affecting the system which is in general both unknown and complex, and depends also on the sample carrier density in a complicated manner. The disorder distribution is characterized by many independent parameters, and therefore, all physical properties of the system, being unique functionals of the disorder distribution, are independent of each other. In particular, the dc conductivity σ , which determines the density-dependent mobility $\mu = \sigma/ne$ where n is the 2D carrier density (and e the magnitude of electron charge), is determined by essentially an integral over the second moment of the disorder distribution whereas other physical properties (i.e., the quality, although there could, in principle, be many independent definitions of sample quality depending on independent experimental measurements of interest) could be determined by other functionals of the disorder distribution. Thus, for any realistic disorder distribution, we do not expect any unique relationship between mobility and quality, and it should be possible, in principle, for samples of different mobility to have the same quality or vice versa (i.e., samples of different quality to have the same 2D mobility).

There is still the vague qualitative expectation, however, that if the sample mobility is enhanced by improving the sample purity (i.e., suppressing disorder), then this should automatically also improve the sample quality (perhaps not necessarily by the same quantitative factor) since reduced disorder should enhance quality. We will show below that this may not always be the case

since mobility and quality (for a specific property) may be sensitive to completely different aspects of disorder and therefore enhancing mobility by itself may do nothing to improve quality in some situations. On the other hand, when mobility and quality are both determined by exactly the same microscopic disorder in the sample, increasing (decreasing) one would necessarily improve (suppress) the other although we will see later in this work [see Fig. 1(c)], for example) this is not necessarily true if mobility and quality are both determined by remote dopant scattering arising from long-range Coulomb disorder.

It is important to emphasize a subtle aspect of the mobility/quality dichotomy with respect to experimental samples. Theoretically, we can consider a hypothetical system with continuously variable disorder (i.e., the parameters characterizing the disorder distribution such as the quenched charged impurity density and their strength as well as their spatial locations including possible spatial correlations in the impurity distribution can all be varied at will). Experimentally, however, the situation is qualitatively different. One does not typically change the impurity distribution in a sample in a controlled manner and make measurements as a function of disorder distribution. Experimentally, measurements are made in *different* samples and compared, and in such a situation there is no reason to expect two samples with identical mobility at some specified carrier density to have identical impurity distributions. The impurity distributions in different samples can be considered to be identical only if the full density-dependent mobility $\mu(n)$, or equivalently the density-dependent conductivity $\sigma(n)$, are identical in all the samples. Such a situation of course never happens in practice, and typically when experimental mobilities in different samples are quoted to be similar in magnitudes, one is referring to either the maximum mobility (occurring typically at different carrier densities in different samples) or the mobility at some fixed high carrier density (and not over a whole range of carrier density). If two samples happen to have the same maximum mobility or the same mobility at one fixed density, there is no reason to expect them to have the same quality with respect to all experimental properties at arbitrary densities. This obvious aspect of mobility versus quality dichotomy has not much been emphasized in the literature.

It should be clear from the discussion above that in the ‘trivial’ (and experimentally unrealistic in 2D semiconductor structures) situation of the system having just one type of impurities uniquely defining the applicable impurity distribution, both mobility and quality, by definition, would be determined by exactly the same impurity configuration since there is just one set of impurities by construction. Such a situation is, in fact, common in 3D semiconductors where the applicable disorder is almost always described by a random uniform background of uncorrelated quenched charged impurity centers, which can be uniquely characterized by a single 3D impurity density n_i . Obviously, in this situation both mobility and

quality are uniquely defined by n_i , and thus increasing (decreasing) n_i would decrease (increase) both mobility and quality (although not necessarily by the same quantitative factor since mobility and quality are likely to be different functions of system parameters in general). In such a simplistic situation, mobility and quality are likely to monotonically connected by a unique relationship, and hence enhancing system mobility should always improve the system quality since the same disorder determines both properties. In fact, this simple situation always applies if the mobility/quality are both limited by purely short-range disorder in the system.

By contrast, 2D semiconductor structures almost always have several qualitatively distinct disorder mechanisms affecting transport and other properties arising from completely different physical origins. For example, it is well-known⁹ that Si-MOSFETs have at least three different operational disorder mechanisms: random charged impurities in the insulating SiO₂ oxide layer, in the bulk Si itself, and random short-range surface roughness at the Si-SiO₂ interface. There may still be other distinct scattering mechanisms associated with still different disorder sources in Si-MOSFETs such as neutral defects or impurities, making the whole situation quite complex. In MOSFETs, low-density carrier transport is controlled by long-range charged impurity scattering whereas the high-density carrier transport is controlled by short-range surface roughness scattering, and this dichotomy may very well lead to situations where a measured electronic property (i.e., “quality”) does not necessarily correlate with the high-density maximum mobility of the system. In high-mobility 2D GaAs-AlGaAs-based systems of interest in the current work, there are at least six distinct scattering mechanisms of varying importance arising from different physical sources of disorder in the system. These are: Unintentional background charged impurities in the 2D GaAs conducting layer; remote dopant impurities in the insulating AlGaAs layer (which are necessary for introducing 2D carriers to form the 2DEG); short-range interface roughness at the GaAs-AlGaAs interfaces; short-range disorder in the insulator AlGaAs layer arising from alloy disorder (and neutral defects); unintentional background charged impurities in the insulating AlGaAs barrier regime; random charged impurities at the GaAs-AlGaAs interface. This is obviously a highly complex situation where the complete disorder distribution will have many independent parameters, and in general, there is no reason to expect a unique relationship between mobility and quality since mobility could be dominated by one type of disorder (e.g., background unintentional charged impurities) and quality may be dominated by a different type of disorder (e.g., remote dopants far from the 2D layer).

It is clear from the above discussion that the minimal disorder model capable of capturing the mobility versus quality dichotomy in high-quality 2D semiconductor structures is a “2-impurity” model with one type of impurity right in the 2D layer itself (arising, for exam-

ple, from the unintentional background impurities in the system) and the other type of impurity being a remote layer separated by a distance ‘ d_R ’ from the 2D electron layer. This minimal 2-impurity model is characterized by three independent parameters: n_R and d_R , denoting respectively the 2D charged impurity density in the remote dopant layer separated by a distance d_R from the 2D carriers, and n_B , the 2D impurity density corresponding to the unintentional background impurities in the 2D layer with $d = 0$. It is easy to go beyond this minimal model and consider the remote impurities to be distributed over a finite distance (rather than simply being placed in a δ -function like layer located at a distance d_R from the 2D system) or assume the background impurities to be distributed three-dimensionally (rather than in a 2D plane at $d = 0$), but such extensions do not modify any of our qualitative conclusion in the current paper as we have explicitly checked numerically. Also, we discuss our theory assuming a strict 2D model (with zero thickness) for the electron layer because the finite quasi-2D layer thickness has no qualitative effect on the physics of quality versus mobility being discussed in this work. Many of our numerical results are, however, obtained by incorporating the appropriate quantitative effects of the quasi-2D layer thickness of the 2D carriers in calculating the mobility and the quality of the system within the 2-impurity model.

One last issue we need to discuss in the Introduction is the question of how to define the sample quality since it is obviously not a unique property and depends on the specific experiment being carried out. In order to keep things both simple and universal, we have decided to use the level broadening or the Dingle temperature as a measure of the sample quality. The level broadening Γ is defined as $\Gamma = \hbar/2\tau_q$, where τ_q is the quantum scattering time (or the single particle relaxation time) in contrast to the mobility scattering time (or the transport relaxation time) τ_t which defines the conductivity (σ) or the carrier mobility (μ) through $\sigma = ne^2\tau_t/m$ or $\mu = ne\tau_t$. In general, $\tau_q \leq \tau_t$ with the equality holding for purely short-range disorder scattering. Thus, for pure s -wave δ -function short-range scattering model, mobility and quality are identical for obvious reasons. For a strict 1-impurity model of underlying sample disorder, mobility (τ_t) and quality (τ_q) are both affected by the same impurity density, and hence they must behave monotonically (but not necessarily identically) with changing disorder, i.e., if the impurity density is decreased (increased) at a fixed carrier density, both mobility and quality must increase (decrease) as well.

As emphasized already, however, the 2-impurity model (near and far impurities or background and remote impurities) introduces a new element of physics by allowing for the possibility that mobility and quality could possibly be affected more strongly by different types of disorder, for example, mobility (quality) could be dominated by near (far) impurities in the 2-impurity disorder model, thus allowing, in principle, the possibility of mobility and

quality being completely independent physical properties of realistic 2D semiconductor samples at least in some situations. We find this situation to be quite prevalent in very high-mobility 2D semiconductor structures where the mobility (quality) seems to be predominately determined by near (far) impurities. In low mobility samples, on the other hand, the situation is simpler and both mobility and quality are typically determined by the same set of impurities (usually the charged impurities close to the 2D layer itself).

In section II we describe our model giving the theoretical formalism and equations for the 2-impurity model. In section III we provide detailed results for mobility and quality along with discussion. We conclude in section IV with a summary of our findings along with a discussion of our approximations and of the open questions.

II. MODEL AND THEORY

We assume a 2D electron (or hole) system at $T = 0$ located at the $z = 0$ plane with the 2D layer being in the x-y plane in our notation. The 2D system is characterized entirely by a carrier effective mass (m) defining the single-particle kinetic energy ($E_{\mathbf{k}} = \hbar^2 k^2 / 2m$ with \mathbf{k} as the 2D wave vector), a background lattice dielectric constant (κ) defining the 2D Coulomb interaction [$V(\mathbf{q}) = 2\pi e^2 / \kappa q$ with \mathbf{q} as the 2D wave vector], and a 2D carrier density (n).

As discussed in the Introduction, we use a minimal 2-impurity model for the static disorder in the system characterized by three independent parameters: n_R , d_R , n_B . Here n_R (n_B) is the effective 2D charged impurity density for the remote (background) impurities with the remote (background) impurities being distributed randomly in the 2D x-y plane at a distance ‘ d ’ from the 2D electron system in the z -direction with $d = d_R$ (0) for the remote (background) impurities. We assume the random quenched charged impurities to all have unit strength (i.e., having an elementary charge of $\pm e$ each) with no loss of generality.

Our model thus has four independent parameters with dimensions of length: $n^{-1/2}$, $n_R^{-1/2}$, $n_B^{-1/2}$, d_R . In addition to these (experimentally variable) parameters, we also have m and κ defining the material system which is fixed in all samples for a given material. In principle, two additional materials parameters should be added to describe the most general situation, namely, the spin (g_s) and the valley (g_v) degeneracy, but we assume $g_s = 2$ and $g_v = 1$ throughout the current work (and for all our numerical results) since our interest here is entirely focussed on high-mobility n- and p-GaAs 2D systems where the mobility/quality dichotomy has mostly been discussed. Additional experimentally relevant (but, theoretically non-essential) parameters, such as a finite width of the 2D electron system (instead of the strict 2D limit) and/or a 3D distribution of the random impurities (instead of the 2D distribution assumed above), are straight-

forward to include in the model and are not discussed further in details.

The mobility (quality) is now simply defined by the characteristic scattering time τ_t (τ_q) as given by the following equations in our leading-order transport theory (i.e., Boltzmann transport plus Born approximation for scattering):

$$\frac{1}{\tau_t(k)} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}'} \sum_l \int_{-\infty}^{\infty} dz N_i^{(l)}(z) |u_{\mathbf{k}-\mathbf{k}'}(z)|^2 \times (1 - \cos \theta) \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}), \quad (1)$$

and

$$\frac{1}{\tau_q(k)} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}'} \sum_l \int_{-\infty}^{\infty} dz N_i^{(l)}(z) |u_{\mathbf{k}-\mathbf{k}'}(z)|^2 \times \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}), \quad (2)$$

where $N_i^{(l)}(z)$ is the 3D impurity distribution for the l -th kind of disorder in the system, and $u_{\mathbf{q}}(z)$ is the screened electron-impurity Coulomb interaction given by:

$$u_{\mathbf{q}}(z) = \frac{V_{\mathbf{q}}(z)}{\varepsilon(q)} = \frac{2\pi e^2}{\kappa q} \frac{e^{-qz}}{\varepsilon(q)}, \quad (3)$$

with $\varepsilon(q)$ being the static RPA dielectric function for the 2D electron system. We note that $V_q(z) = V(q)e^{-qz} = \frac{2\pi e^2}{\kappa q} e^{-qz}$ is simply the 2D Fourier transform of the 3D $1/r$ Coulomb potential, which explicitly takes into account the fact that a spatial separation of ‘ z ’ may exist between the 2D electron layer and the charged impurities. The 2D static RPA dielectric function or the screening function is given by

$$\varepsilon(q) = 1 + \frac{2\pi e^2}{\kappa q} \Pi(q), \quad (4)$$

where the static 2D electronic polarizability function $\Pi(q)$ is given by

$$\Pi(q) = N_F \left[1 - \theta(q - 2k_F) \sqrt{1 - (2k_F/q)^2} \right], \quad (5)$$

where $N_F = m/\pi\hbar^2$ and $\theta(x) = 0$ (1) for $x < 0$ ($x > 0$) is the Heaviside step (or theta) function and the 2D Fermi wave vector k_F is determined by the 2D carrier density through the formula $k_F = (2\pi n)^{1/2}$. We note that the 2D Fermi energy (E_F) is given by $E_F = \hbar^2 k_F^2 / 2m = \pi\hbar^2 n / m$. The RPA screening function $\varepsilon(q)$ can be expressed in the convenient form

$$\varepsilon(q) = 1 + q_s/q, \quad (6)$$

which is exactly equivalent to Eqs. (4) and (5) if we define the 2D screening wave vector q_s to be

$$q_s = q_{TF} \left[1 - \theta(q - 2k_F) \sqrt{1 - (2k_F/q)^2} \right], \quad (7)$$

where the 2D Thomas-Fermi wave vector q_{TF} is defined to be

$$q_{TF} = 2me^2 / (\kappa\hbar^2). \quad (8)$$

We note that (1) the Thomas-Fermi wave vector q_{TF} is proportional to the 2D density of states at the Fermi energy $N_F = m/\pi\hbar^2$ (and is inversely proportional to the effective background lattice dielectric constant κ), and (2) screening is constant in 2D for $0 \leq q \leq 2k_F$ [see Eq. (7)] because of the constant energy-independent 2D electronic density of states. Since the δ -functions (necessary for energy conservation during the impurity-induced elastic scattering of an electron from the momentum state $|\mathbf{k}\rangle$ to the momentum state $|\mathbf{k}'\rangle$ with a net wave vector transfer of $\mathbf{q} = \mathbf{k} - \mathbf{k}'$) in Eqs. (1) and (2) restrict the wave vector transfer $0 \leq q \leq 2k_F$ range (this is simply because we are at $T = 0$ so that the maximum possible scattering wave vector is $2k_F$ corresponding to the pure back-scattering of an electron from $+k_F$ to $-k_F$ while obeying energy conservation), the relevant screening wave vector for our problem is purely the Thomas-Fermi wave vector $q_s = q_{TF}$ as follows from Eq. (7) for $q \leq 2k_F$.

We can, therefore, rewrite Eq. (3) as

$$u_q(z) = \frac{2\pi e^2}{\kappa(q + q_{TF})} e^{-qz}, \quad (9)$$

giving the effective screened Coulomb interaction (in the 2D momentum space) between an electron in the 2D layer and a random quenched charged impurity located a distance ‘ z ’ away.

Finally, our 2-impurity disorder model is given by

$$N_i(z) = n_R \delta(z - d_R) + n_B \delta(z), \quad (10)$$

with three independent parameters n_R , d_R , and n_B completely defining the underlying disorder. We note that writing the second term in Eq. (10) as $n_B \delta(z - d_B)$, and thus introducing an additional length parameter into the model, is completely unnecessary since, as we will see below, the physics of the mobility versus quality duality in high-quality 2D structures is entirely dominated by scattering from near impurities (controlling mobility) and far impurities (controlling quality), which allows us to put $d_B = 0$ in the minimal model (thus making the near impurities very near indeed). Putting Eqs. (3) – (10) in Eqs. (1) and (2), we can combine them into a single 2D integral, obtaining

$$\frac{1}{\tau_{t,q}} = \frac{2\pi}{\hbar} \left(\frac{2\pi e^2}{\kappa} \right)^2 \int \frac{d^2 k'}{(2\pi)^2} \frac{\delta(E_{\mathbf{k}} - E_{\mathbf{k}'})}{(q_{TF} + |\mathbf{k} - \mathbf{k}'|)^2} f_{t,q}(\theta) \times \left\{ n_R e^{-2|\mathbf{k} - \mathbf{k}'|d_R} + n_B \right\}, \quad (11)$$

where $f_t(\theta) = 1 - \cos \theta_{kk'}$ and $f_q(\theta) = 1$. For completeness, we mention that $E_{\mathbf{k}} = \hbar^2 k^2 / 2m$.

Before proceeding further, we emphasize that the “only” difference between mobility (i.e., τ_t) and quality (i.e., τ_q) is the appearance (or not) of the angular factor $(1 - \cos \theta)$ inside the double integral in Eq. (11) for τ_t (τ_q). This arises from the fact that the mobility or the conductivity is unaffected by forward scattering (i.e., $\theta \approx 0$ or $\cos \theta \approx 1$) whereas the single-particle level-broadening ($\Gamma \sim \hbar/\tau_q$) is sensitive to scattering through

all angles. Technically, the $(1 - \cos \theta)$ factor arises from the impurity scattering induced vertex correction in the 2-particle current-current correlation function representing the electrical conductivity whereas the single-particle scattering rate (τ_q^{-1}) is given essentially by the imaginary part of the impurity scattering induced 1-particle electronic self-energy which does not have any vertex correction in the leading-order impurity scattering strength. The absence (presence) of the vertex correction in τ_q^{-1} (τ_t^{-1}) makes the relevant scattering rate sensitive (insensitive) to forward scattering, leading to a situation where τ_q and τ_t could be very different from each other if forward (or small angle) scattering is particularly important in a system as it would be for long range disorder potential.

This could happen in 3D systems if the scattering potential is strongly spatially asymmetric (or non-spherical) for some reason. It was pointed out¹⁰ a long time ago and later experimentally verified¹¹ that such a strongly non-spherically symmetric scattering potential arises naturally in 2D modulation-doped structures from random charged impurities placed very far ($k_F d \gg 1$) away from the 2D electron system due to the influence of the exponential e^{-qd} factor in the Coulomb potential which restricts much of the scattering to $q \ll 1/z$, thus exponentially enhancing the importance of small-angle (i.e., small scattering wave vector) scattering. This then leads to $\tau_t \gg \tau_q$ for scattering dominated by remote dopants (the two scattering times could differ by more than two orders of magnitude in high-mobility modulation-doped structures where $k_F d_R \gg 1$ is typically satisfied due to the far-away placement of the remote dopants in order to minimize large-angle resistive scattering processes) in 2D systems, but for the unintentional background impurities, which always satisfy $k_F d_B \ll 1$ by definition (since $d_B \approx 0$), the two scattering times are approximately equal. We note that the finite layer thickness of the 2D system puts a lower bound on how small d_B can be, but this has no qualitative significance for our consideration.

Now, we immediately realize the crucial relevance of the 2-impurity model in distinguishing mobility (i.e., τ_t) and quality (i.e., τ_q) in 2D systems since it now becomes possible for one type of disorder (e.g., remote impurities) to control the quality (i.e., τ_q) and the other type to control the mobility (i.e., τ_t). Of course, whether such a distinction actually applies to a given situation or not depends entirely on the details of the sample parameter (i.e., the specific values of n_R , d_R , n_B) as well as the carrier density n , but the possibility certainly exists for d_R to be large enough so that the remote impurity scattering is almost entirely small-angle scattering (thus only adversely affecting τ_q in an appreciable way) whereas the background impurity scattering determines τ_t . If this happens, then mobility (τ_t) and quality (τ_q) could very well be very different in 2D samples, and may have little to do with each other.

To bring out the above physical picture explicitly, we

now provide some analytical calculations for the integrals in Eq. (11) defining mobility (τ_t) and quality (τ_q). We rewrite Eq. (11) as

$$\tau_{t,q}^{-1} = I_{t,q}^{(R)} + I_{t,q}^{(B)}, \quad (12)$$

where

$$I_{t,q}^{(R)} = n_R V_0 \frac{2\pi}{\hbar} \int \frac{d^2 k'}{(2\pi)^2} \frac{\delta(E_{\mathbf{k}} - E_{\mathbf{k}'})}{(q_{TF} + |\mathbf{k} - \mathbf{k}'|)^2} \times f_{t,q}(\theta) e^{-2|\mathbf{k} - \mathbf{k}'|d_R}, \quad (13)$$

and

$$I_{t,q}^{(B)} = n_B V_0 \frac{2\pi}{\hbar} \int \frac{d^2 k'}{(2\pi)^2} \frac{\delta(E_{\mathbf{k}} - E_{\mathbf{k}'})}{(q_{TF} + |\mathbf{k} - \mathbf{k}'|)^2} f_{t,q}(\theta), \quad (14)$$

where $V_0 = (2\pi e^2/\kappa)^2$. Eqs. (13) and (14) can be rewritten in dimensionless forms as

$$I_{t,q}^{(R)} = \left(\frac{mn_R V_0}{2\pi \hbar^3 k_F^2} \right) \int_0^1 dx \frac{g_{t,q}(x) e^{-2xa_R}}{\sqrt{1-x^2}(x+s)^2} \quad (15)$$

and

$$I_{t,q}^{(B)} = \left(\frac{mn_B V_0}{2\pi \hbar^3 k_F^2} \right) \int_0^1 dx \frac{g_{t,q}(x)}{\sqrt{1-x^2}(x+s)^2} \quad (16)$$

where $a_R \equiv 2k_F d_R$ and $s = q_{TF}/2k_F$, and $g_t(x) = 2x^2$, $g_q(x) = 1$.

To proceed further analytically, we now make the assumption that, by definition, the remote dopants are far enough that the condition $a_R = 2k_F d_R \gg 1$ is satisfied. We note that for any arbitrarily large value of d_R , this ‘remote’ impurity condition breaks down at a low enough carrier density such that $n \lesssim 1/8\pi d_R^2$. Thus, the distinction between ‘far’ and ‘near’ impurities in our 2-impurity model starts disappearing at very low carrier density. Putting $a_R \gg 1$ as well as $s = q_{TF}/2k_F \gg 1$ we can obtain the following asymptotic expressions for $I_{t,q}^{(R)}$

$$I_t^{(R)} = \left(\frac{mn_R}{2\pi \hbar^3 k_F^2} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \left(\frac{2}{3s^2} \right) \frac{1}{a_R^3}, \quad (17)$$

$$I_q^{(R)} = \left(\frac{mn_R}{2\pi \hbar^3 k_F^2} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \frac{1}{a_R s^2}, \quad (18)$$

$$I_t^{(B)} = \left(\frac{mn_B}{2\pi \hbar^3 k_F^2} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \frac{2\pi}{s^2}, \quad (19)$$

$$I_q^{(B)} = \left(\frac{mn_B}{2\pi \hbar^3 k_F^2} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \frac{2\pi}{s^2}, \quad (20)$$

with $\tau_{t,q}^{-1} = I_{t,q}^{(R)} + I_{t,q}^{(B)}$. Equations (17) – (20) provide us with approximate analytical expressions for the contributions $I_{t,q}^{(R)}$ and $I_{t,q}^{(B)}$ by the remote and the background scattering respectively to the transport and quantum scattering rates. We note that in these asymptotic

limits ($a_R \gg 1$ and $s \gg 1$)

$$\frac{I_t^{(B)}}{I_t^{(R)}} = 3\pi a_R^3 \frac{n_B}{n_R}, \quad (21)$$

and

$$\frac{I_q^{(R)}}{I_q^{(B)}} = \frac{1}{2\pi a_R} \frac{n_R}{n_B}. \quad (22)$$

In addition,

$$I_q^{(R)}/I_t^{(R)} = 3a_R^2/2, \quad (23)$$

$$I_q^{(B)}/I_t^{(B)} = 1. \quad (24)$$

We note, therefore, that $I_q^{(R)} \gg I_t^{(R)}$ (since $a_R \gg 1$) as is already well established, and that $I_t^{(B)} > I_t^{(R)}$ if $3\pi a_R^3 > n_R/n_B$, whereas $I_q^{(R)} > I_q^{(B)}$ if $n_R/n_B > 2\pi a_R$. Consistency demands that $3\pi a_R^3 \gg 2\pi a_R$, i.e., $a_R^2 \gg 2/3$, which is guaranteed since $a_R \gg 1$.

It is thus possible for the B -scatterers to dominate τ_t^{-1} , i.e., $I_t^{(B)} \gg I_t^{(R)}$, and R -scatterers to dominate τ_q^{-1} , i.e., $I_q^{(R)} \gg I_q^{(B)}$ if the following conditions are both satisfied

$$\begin{aligned} 3\pi a_R^3 &\gg n_R/n_B, \\ 2\pi a_R &\ll n_R/n_B. \end{aligned} \quad (25)$$

The two inequalities defined by Eq. (25) are not mutually exclusive if $n_R \gg n_B$ and

$$n_R/2\pi n_B \gg a_R \gg (n_R/3\pi n_B)^{1/3}. \quad (26)$$

It is easy to see the Eq. (26) is consistent as long as $n_R \gg 5.13n_B$, a perfectly reasonable scenario! In fact, we expect this condition to be extremely well-satisfied in high-quality 2D systems where n_B is very small, but $n_R \approx n$ due to modulation doping and charge neutrality.

The above analytic considerations lead to the conclusion that it is possible for τ_t^{-1} to be dominated by background impurities, and at the same time for τ_q^{-1} to be dominated by the remote impurities provided the necessary conditions $a = 2k_F d_R \gg 1$ and $n_R \gg 5.13n_B$ are obtained. We emphasize that these are only *necessary* conditions, and not sufficient conditions. Whether the 2-impurity model indeed leads to mobility (i.e., τ_t) and quality (i.e., τ_q) being controlled by physically distinct disorder mechanisms in real 2D semiconductor structures can only be definitely established through explicit numerical calculations of $\tau_{t,q}^{-1}$ for specific disorder configurations, which we do in the next section of this article. The general analytical theory developed above also explicitly shows that there can be no mobility/quality dichotomy if there is only one type of disorder mechanism operational in the sample since both quality and mobility will then be controlled by exactly the same disorder parameters (unlike the situation discussed above).

It may be useful to write a full analytical expression for τ_t^{-1} and τ_q^{-1} (in the $a_R \gg 1$ limit) combining all the expressions given above to show how the disorder parameters n_R , d_R , and n_B enter the expressions for mobility and quality:

$$\tau_t^{-1} = I_t^{(R)} + I_t^{(B)} = A_t^{(R)} n_R / a_R^3 + A_t^{(B)} n_B, \quad (27)$$

$$\tau_q^{-1} = I_q^{(R)} + I_q^{(B)} = A_q^{(R)} n_R / a_R + A_q^{(B)} n_B, \quad (28)$$

where

$$A_t^{(R)} = \left(\frac{m}{2\pi\hbar^3} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \left(\frac{8}{3q_{TF}^2} \right)$$

$$A_t^{(B)} = \left(\frac{m}{2\pi\hbar^3} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \left(\frac{8\pi}{q_{TF}^2} \right), \quad (29)$$

$$A_q^{(R)} = \left(\frac{m}{2\pi\hbar^3} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \left(\frac{4}{q_{TF}^2} \right)$$

$$A_q^{(B)} = \left(\frac{m}{2\pi\hbar^3} \right) \left(\frac{2\pi e^2}{\kappa} \right)^2 \left(\frac{8\pi}{q_{TF}^2} \right). \quad (30)$$

Equations (27) and (28) immediately lead to the approximate sufficient conditions for mobility and quality to be controlled by background and remote impurities, respectively:

$$n_B \gg n_R / a_R^3, \text{ i.e., } n_R \ll n_B a_R^3, \quad (31)$$

and

$$n_R / a_R \gg n_B, \text{ i.e., } n_R \gg n_B a_R. \quad (32)$$

Since $a_R \gg 1$ by definition, the two conditions, Eqs. (31) and (32), can simultaneously be satisfied if

$$a_R^3 \gg n_R / n_B \gg a_R, \quad (33)$$

with $a_R \gg 1$. Equation (33) gives the sufficient condition for the existence of a mobility/quality dichotomy in 2D semiconductor structures. Since the unintentional background charged impurity concentration is typically very low in high-mobility 2D GaAs structures and since the remote charged dopant density is typically (at least) equal to the carrier density, the condition $a_R^3 \gg n_R / n_B \gg a_R \gg 1$ can certainly be satisfied in some 2D samples (but obviously not in all samples). For example, a typical modulation-doped high mobility 2D GaAs-Al_xGa_{1-x}As structure may have $n \approx n_R \approx 3 \times 10^{11} \text{ cm}^{-2}$; $d_R \approx 1000 \text{ \AA}$; $n_B \approx 10^8 \text{ cm}^{-2}$ (corresponding roughly to a 3D bulk background charged impurity density of $3 \times 10^{13} \text{ cm}^{-3}$ for a 300 \AA wide quantum well structure). These system parameters satisfy the constraint defined by Eq. (33) with $k_F d_R \approx 15$, i.e., $a_R \approx 30$; $n_R / n_B \approx 3000$. We emphasize the obvious role of carrier density here, i.e., lowering the carrier density decreases $k_F d$ (and hence a_R), and eventually at low enough carrier density, τ_t^{-1} and τ_q^{-1} are determined by the same disorder parameters!

In concluding this theoretical section, let us consider a concrete numerical example of the mobility/quality dichotomy using two hypothetical samples (1 and 2) with the following realistic sample parameters:

$$\text{Sample 1 : } d_R^{(1)} = 500 \text{ \AA}; n_B^{(1)} = 10^7 \text{ cm}^{-2}$$

$$\text{Sample 2 : } d_R^{(2)} = 1000 \text{ \AA}; n_B^{(2)} = 10^8 \text{ cm}^{-2}. \quad (34)$$

We assume the sample carrier density to be the same in both cases (so that ‘‘an apple-to-apple’’ comparison in being made): $n = 4 \times 10^{11} \text{ cm}^{-2}$. For the purpose of keeping the number of parameters a minimum we assume $n_R = n = 4 \times 10^{11} \text{ cm}^{-2}$ also for both samples. Using the analytical theory developed above (or by direct numerical calculations), we find

$$\tau_t^{(1)} / \tau_t^{(2)} \approx 5; \tau_q^{(1)} / \tau_q^{(2)} \approx 2. \quad (35)$$

This means that sample 1 (with $n_B^{(1)} < n_B^{(2)}$) has a five times higher mobility than sample 2 whereas sample 2 (with $d_R^{(2)} > d_R^{(1)}$) has two times higher ‘quality’ than sample 1, i.e., sample 2 has a single-particle level broadening Γ (or ‘‘Dingle temperature’’) which is half of that of sample 1 although both samples have exactly the same carrier density! This realistic example shows that it is generically possible in the 2-impurity model for $\mu_1 > \mu_2$ and $\Gamma_1 > \Gamma_2$, with the conclusion that a higher mobility does not necessarily ensure a higher quality. We emphasize that (1) this would not be possible within the 1-impurity model, and (2) this conclusion is density dependent – for much lower carrier density, where $k_F d \gg 1$ condition cannot be satisfied for the remote dopants, mobility and quality will again be closely connected since at sufficiently low carrier density, the 2-impurity model effectively reduces to an 1-impurity model.

Before presenting our realistic numerical results for 2D GaAs-AlGaAs structures (including both the quasi-2D finite well-width effect and a 3D distribution of the background unintentional charged impurity distribution within the well) using the ‘near’ and ‘far’ 2-impurity model in the next section (Sec. III), we conclude the current theory section by showing some numerical results for τ_t and τ_q using the idealized 2-impurity model [i.e., Eq. (10)] and the strict 2D model for the semiconductor structure. These results presented in Figs. 1 – 4 explicitly visually demonstrate that τ_t and τ_q cannot be single-valued functions of each other as long as the underlying disorder consists of (at least) two distinct scattering mechanisms as operational within the 2-impurity model. The results shown in Figs. 1 – 4 also serve to establish the validity of the analytical theory we provided above in this section.

In Fig. 1 we show the dependence of the calculated τ_t and τ_q on the individual scattering mechanism (i.e., the near-impurity scattering strength defined by the background impurity concentration n_B or the far-impurity scattering strength defined by either n_R or d_R) assuming that the other mechanism is absent (i.e., just an effective 1-impurity model applies). Results in Fig. 1 should

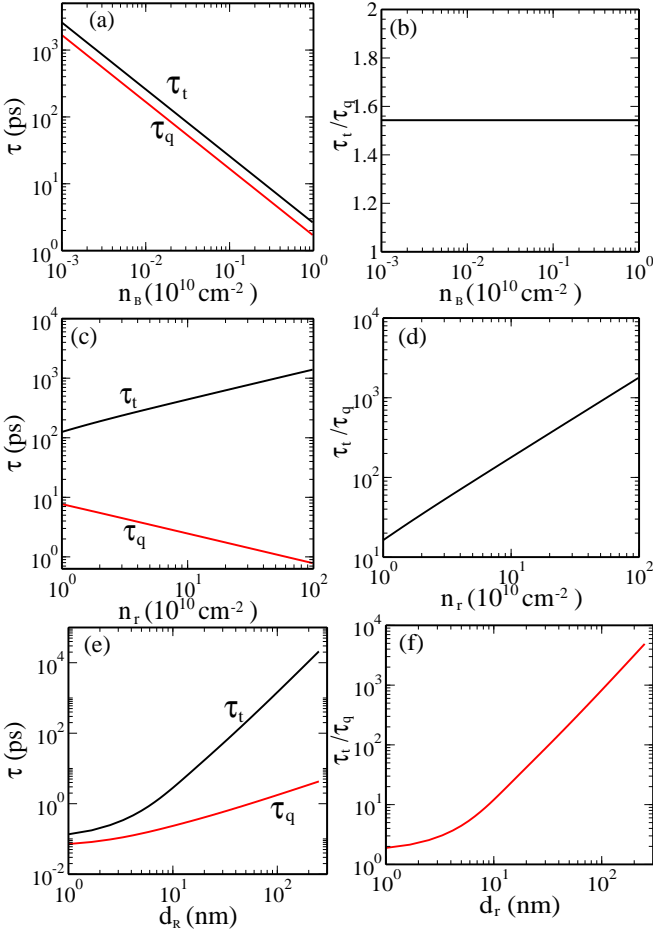


FIG. 1. (a) The scattering times (τ_t and τ_q) and (b) the ratio (τ_t/τ_q) as a function of the background impurity density n_B for $n = 3 \times 10^{11} \text{ cm}^{-2}$ and $n_R = 0$. (c) The scattering times and (d) the ratio (τ_t/τ_q) as a function of the remote impurity density n_R for $n = n_R$, $n_B = 0$, and $d_R = 80 \text{ nm}$. (e) The scattering times and (f) the ratio (τ_t/τ_q) as a function of the impurity location d_r for $n = n_r = 3 \times 10^{11} \text{ cm}^{-2}$ and the background impurity density $n_B = 0$. Here the δ -layer is considered (i.e., $a = 0$).

be compared with the corresponding results in Figs. 2 – 4 where both scattering mechanisms are operational to clearly see that τ_t and τ_q are manifestly not unique functions of each other by any means and a given value of τ_t (or τ_q) could lead to distinct values of τ_q (or τ_t) depending on the details of the disorder distribution. Thus, mobility (τ_t) and quality (τ_q) are not simply connected.

In presenting the results for Figs. 1 – 4 we first note that $\tau_{t,q} = \tau(n, n_B, n_R, d_R)$ even within the simple 2-impurity model. Since the carrier density dependence of τ is not the central subject matter of our interest in the current work (and has been discussed elsewhere by us⁸, we simplify the presentation by assuming $n_R = n$ in Figs. 1 – 4 which also assures a straightforward charge neutrality. This, however, has important implications since the dependence on n_R and n now become com-

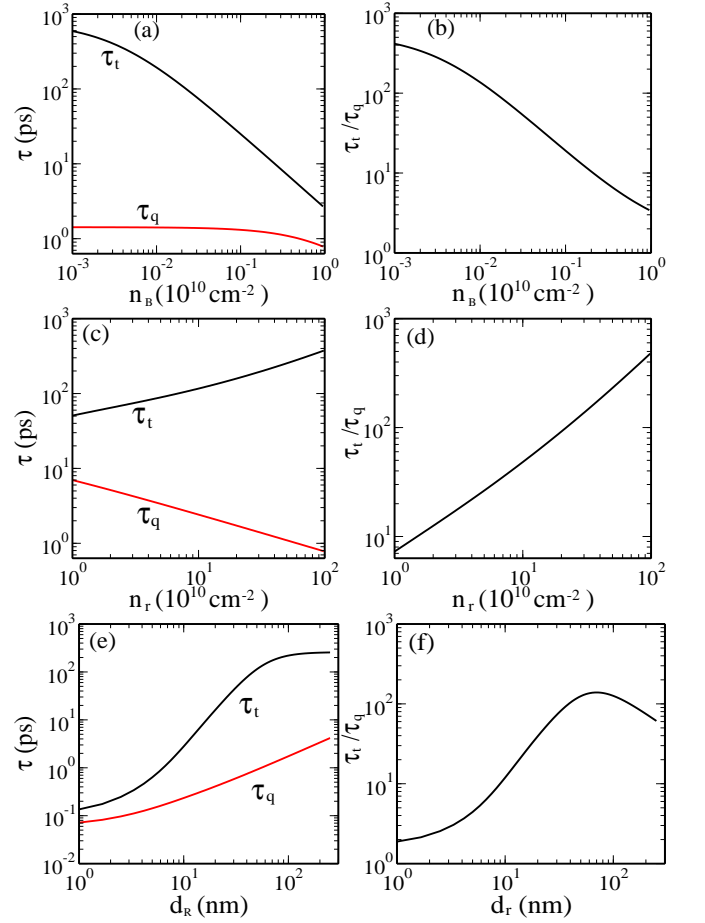


FIG. 2. (a) The scattering times (τ_t and τ_q) and (b) the ratio (τ_t/τ_q) as a function of the background impurity density n_B for $n = n_R = 3 \times 10^{11} \text{ cm}^{-2}$ and $d_R = 80 \text{ nm}$. (c) The scattering times and (d) the ratio (τ_t/τ_q) as a function of the remote impurity density n_R for $n = n_R$, $n_B = 10^8 \text{ cm}^{-2}$, and $d_R = 80 \text{ nm}$. (e) The scattering times and (f) the ratio (τ_t/τ_q) as a function of the impurity location d_r for $n = n_R = 3 \times 10^{11} \text{ cm}^{-2}$ and the background impurity density $n_B = 0$. Here the δ -layer is considered (i.e., $a = 0$).

pounded rather than being independent. Thus the n_R -dependence of τ shown in Figs. 1 – 4 is not the trivial $\tau_{t,q} \sim n_R^{-1}$ behavior (as it is for the n_B -dependence, where $\tau_{t,q} \sim n_B^{-1}$) since the $n_R = n$ condition synergistically combines both n_R and n dependence. We mention that in ungated samples with fixed carrier density, the condition $n = n_R$ is perfectly reasonable, and therefore, the results shown in Figs. 1 – 4 apply to modulation-doped samples with fixed carrier density $n = n_R$.

From Fig. 1, we immediately conclude the obvious: The functional relationship between τ_t and τ_q depends entirely on which disorder parameter is being varied – in fact Figs. 1(a), (c), (e) give three completely distinct functional relationships between τ_t and τ_q depending on whether n_B , n_R , or d_R is being varied, respectively. From the corresponding values of τ_t/τ_q , as shown in Figs. 1(b), (d), (f) respectively we clearly see that τ_t and τ_q varia-

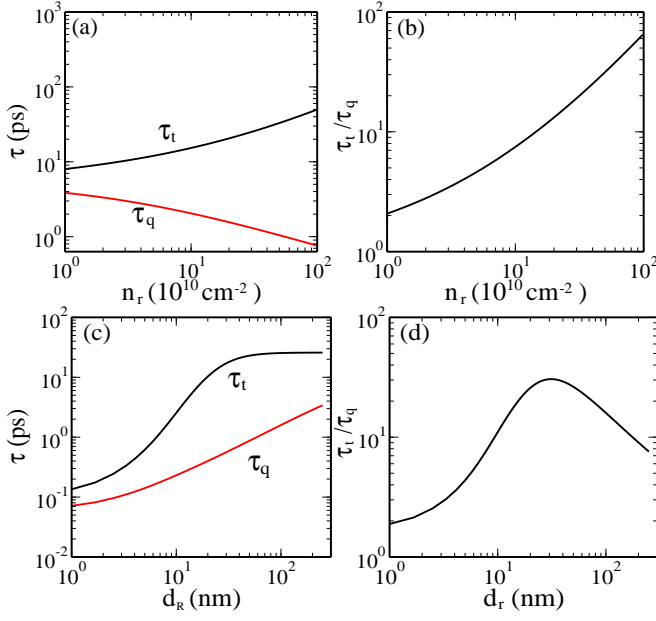


FIG. 3. (a) The scattering times and (b) the ratio (τ_t/τ_q) as a function of the remote impurity density n_R for $n = n_R$, $n_B = 10^9 \text{ cm}^{-2}$, and $d_R = 80 \text{ nm}$. (c) The scattering times and (d) the ratio (τ_t/τ_q) as a function of the impurity location d_R for $n = n_R = 3 \times 10^{11} \text{ cm}^{-2}$ and the background impurity density $n_B = 10^9 \text{ cm}^{-2}$. Here the δ -layer is considered (i.e., $a = 0$).

tions with individual disorder parameters n_B , n_R , d_R are very different indeed. We point out an important qualitative aspect of Fig. 1(c) which has not been explicitly discussed in the literature and which has important implications for the mobility/quality dichotomy. Here the mobility (i.e., τ_t) increases with increasing $n_R = n$, but the quality (i.e., τ_q) decreases with increasing $n = n_R$. This is a peculiar feature of long range Coulomb scattering by remote dopants.

Figs. 2 – 4 explicitly show how the 2-impurity model can very strongly modify the 1-impurity model functional dependence of $\tau_{t,q}$ on the disorder parameters n_B , $n_R (= n)$, and d_R . Clearly, depending on the specific 2D samples, τ_t and τ_q could behave very differently as already established in our analytical theoretical results given above. For example, in contrast to Fig. 1(a), where both τ_t and τ_q decrease monotonically (and trivially as n_B^{-1}) with increasing amount of unintentional background impurity density n_B , Fig. 2(a) shows that τ_q is essentially a constant whereas τ_t decreases with increasing n_B , thus demonstrating a specific example of how the effective quality (i.e. τ_q) remains the same although the effective mobility decreases by more than an order of magnitude due to the variation in the background disorder. Similarly, in Fig. 1(c), increasing the remote dopant separation d_R (keeping $n = n_R$ fixed, and $n_B = 0$) increases both τ_t and τ_q monotonically (with τ_t increasing as d^3 in contrast to τ_q increasing as d for large d), but Fig. 2(e), 3(c), 4(c) show that, depending

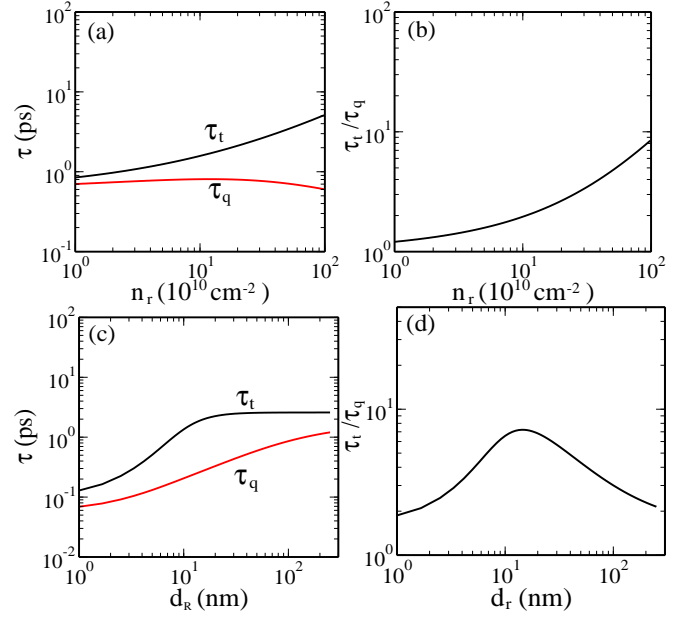


FIG. 4. (a) The scattering times and (b) the ratio (τ_t/τ_q) as a function of the remote impurity density n_R for $n = n_r$, $n_B = 10^{10} \text{ cm}^{-2}$, and $d_R = 80 \text{ nm}$. (c) The scattering times and (d) the ratio (τ_t/τ_q) as a function of the impurity location d_R for $n = n_r = 3 \times 10^{11} \text{ cm}^{-2}$ and the background impurity density $n_B = 10^{10} \text{ cm}^{-2}$. Here the δ -layer is considered (i.e., $a = 0$).

on the background impurity scattering strength, τ_t basically saturates for larger d since it is then dominated by the unintentional background impurities rather than by the remote dopants whereas τ_q continues to be limited by the remote dopants. This leads to an interesting non-monotonicity in τ_t/τ_q as a function of d_R in Figs. 2 – 4 in contrast to Fig. 1(c) where $\tau_t/\tau_q \sim d_R^2$ keeps on increasing forever in the absence of background scattering. The realistic dependence of $\tau_{t,q}$ on n_R (with $n = n_R$) in the presence of fixed n_B and d_R remains qualitatively similar in Figs. 1 – 4 although there could be large quantitative differences, indicating that increasing $n_R (= n)$ would typically by itself tend to enhance (suppress) τ_t (τ_q), but the effect becomes whether for larger (smaller) values of n_B (d_R). This is an important result of our paper.

The analytical and numerical results presented in this section establish clearly that τ_t and τ_q can essentially be independent functions of the disorder parameters in the 2-impurity model, and thus, mobility and quality could, in principle, have little to do with each other in realistic 2D semiconductor structures. We make this point even more explicit by carrying out calculations in experimentally realistic samples in the next section of this article.

III. NUMERICAL RESULTS AND DISCUSSIONS

We begin presenting our realistic numerical results for 2D transport properties (both τ_t or mobility and τ_q or quality) without any reference to the analytical asymptotic theoretical results presented in the last section by showing in Fig. 5 the calculated $T = 0$ mobility (μ), transport scattering time (τ_t) and the single-particle (or quantum) scattering time (τ_q) for a 2D GaAs-AlGaAs sample at a fixed carrier density ($n = 3 \times 10^{11} \text{ cm}^{-2}$) in the presence of two types of disorder: a remote charged impurity sheet ($n_i = 1.5 \times 10^{11} \text{ cm}^{-1}$) placed at a distance ‘ d ’ from the edge of the quantum well (i.e. from the GaAs-AlGaAs interface) and a background 3D charged impurity density (n_{iw}) which is distributed uniformly throughout the inside of the GaAs quantum well (which has a well thickness of 300 Å). Results presented in Fig. 5 involve no approximation other than assuming a uniform random distribution of the quenched charged impurities (2D distribution with a fixed 2D impurity density n_i for the remote impurities placed at a distance d from the quantum well and 3D distribution with a variable 3D impurity density n_{iw} for the unintentional background impurities inside quantum well) as we include the full quantitative effect of the quasi-2D nature of the quantum well width in the calculation and calculate all the integrals [Eqs. (1) and (2)] for τ_t^{-1} and τ_q^{-1} numerically exactly. Of course, our basic theory is a leading-order theory in the impurity scattering strength which should be an excellent approximation at the high carrier density of interest in the current work where our focus is on very low-disorder and high-quality 2D semiconductor systems. The use of the realistic 3D background impurity distribution is easily reconciled with our minimal model in section II by using: $n_i = n_R$, $d_R = d + a/2$ and $n_B \approx n_{iw}a$, where a ($= 300 \text{ Å}$ in Fig. 5) is the quantum well-width.

Results in Fig. 5 are quite revealing of the physics discussed already in section II. First, we clearly see in Fig. 5(a) the trend that for large (small) d , the mobility is determined by the background (remote) impurities, and hence for $d = 800$ (100) Å, the mobility depends strongly (weakly) on the background impurity density (until it becomes very large, leading to $\mu < 10^6 \text{ cm}^2/\text{Vs}$ which is no longer a high-quality situation). In particular, for $d = 800 \text{ Å}$, $\mu^{-1} (\propto \tau_t^{-1}) \propto n_{iw}$ approximately, implying that τ_t^{-1} is dominated almost entirely (for $d = 800 \text{ Å}$) by the background impurities in the quantum well. By contrast, for $d = 100 \text{ Å}$, μ is almost independent of n_{iw} for $n_{iw} \lesssim 10^{15} \text{ cm}^{-3}$ (corresponds to $n_B \sim 3 \times 10^9 \text{ cm}^{-2}$), indicating that τ_t^{-1} is dominated almost entirely by the “remote” dopant scattering. In Fig. 5(a) we focus on the interesting $d = 800 \text{ Å}$ situation where the 2-impurity model might apply – obviously, for $d = 100 \text{ Å}$, the remote dopants dominate both τ_t^{-1} and τ_q^{-1} rendering the 2-impurity model inapplicable since $k_F d < 1$ for both “remote” and “background” impurities for small ‘ d ’.

In Fig. 5(b) we show as a function of background dis-

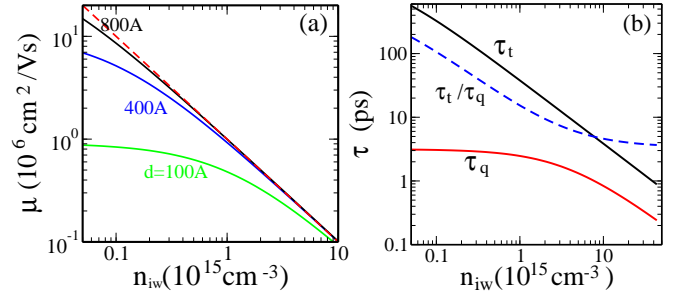


FIG. 5. (a) Calculated mobilities as a function of background 3D charged impurity density n_{iw} for fixed remote impurity density, $n_i = 1.5 \times 10^{11} \text{ cm}^{-2}$, and electron density, $n = 3 \times 10^{11} \text{ cm}^{-2}$. The red dashed line indicates $\mu \sim n_{iw}$. The numbers indicate the location of remote impurities which is measured from the interface of quantum well. (b) The calculated scattering times and their ratio τ_t/τ_q as a function of background impurity density using the same parameters of (a) and $d = 800 \text{ Å}$. The dashed line represents the ratio of τ_t to τ_q . A quantum well with the thickness of $a = 300 \text{ Å}$ is used in this calculation.

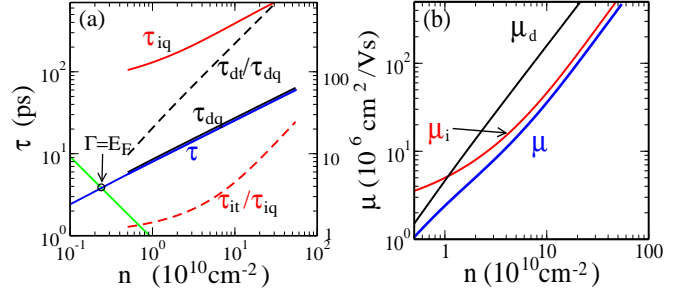


FIG. 6. (a) Calculated scattering times and (b) mobilities as a function of carrier density. Here τ_{iq} , μ_i (τ_{dq} , μ_d) indicate the single particle relaxation time and mobility due to interface impurities at $d = 0$ (remote impurities at finite $d = 800 \text{ Å}$), respectively, and $\tau = (\tau_{iq}^{-1} + \tau_{dq}^{-1})^{-1}$, $\mu = (\mu_i^{-1} + \mu_d^{-1})^{-1}$. The Green line indicates $\tau_q = 0.92/n$ in units of ps with n measured by 10^{10} cm^{-2} . The crossing point between green line and blue line represents $\Gamma = E_F$. The following parameters are used: $n_i(d = 0) = 10^8 \text{ cm}^{-2}$, $n_d(d = 800 \text{ Å}) = 10^{10} \text{ cm}^{-2}$ and quantum well width $a = 300 \text{ Å}$. At a density $n = 10^{11} \text{ cm}^{-2}$ we have $\mu = 35.7 \times 10^6 \text{ cm}^2/\text{Vs}$ and $\tau_q = 25.5 \text{ ps}$. The critical density (i.e. the crossing point) is $n_c = 0.24 \times 10^{10} \text{ cm}^{-2}$.

order the calculated τ_q (as well as τ_t) for $d = 800 \text{ Å}$, and it is clear that for $5 \times 10^{13} \text{ cm}^{-3} < n_{iw} < 2 \times 10^{15} \text{ cm}^{-3}$ (i.e., over a factor of 40 increase in the background disorder!) τ_q (i.e., quality) remains almost a constant whereas τ_t (i.e., mobility) decreases approximately by a factor of 40 in this regime. Combining Figs. 5(a) and (b), we then conclude that there could be an infinite series of samples, where the mobility decreases from $\sim 20\text{--}40 \times 10^6 \text{ cm}^2/\text{Vs}$ to below $10^6 \text{ cm}^2/\text{Vs}$ as n_{iw} increases from $5 \times 10^{13} \text{ cm}^{-3}$ to $2 \times 10^{15} \text{ cm}^{-3}$, with all of them having essentially the same quality as characterized by the quantum scattering time $\tau_q \sim 3 \text{ ps}$, corresponding to a quantum level broadening of $\Gamma \sim \hbar/2\tau_q \sim 0.1 \text{ meV}$. Results shown in Fig. 5,

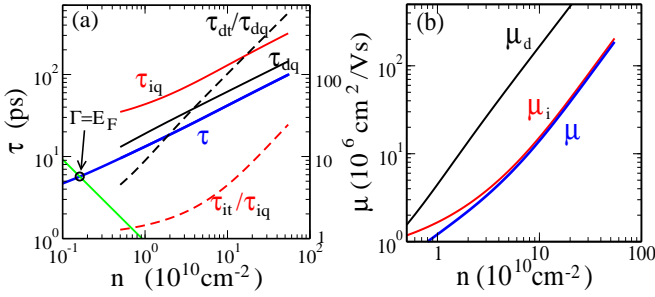


FIG. 7. The same as Fig. 6 with following parameters: $n_i(d=0) = 3 \times 10^8 \text{ cm}^{-2}$, $n_d(d=500\text{\AA}) = 3 \times 10^9 \text{ cm}^{-2}$, and quantum well width $a = 300 \text{\AA}$. At a density $n = 10^{11} \text{ cm}^{-2}$ we have $\mu = 14.1 \times 10^6 \text{ cm}^2/\text{Vs}$ and $\tau_q = 42 \text{ ps}$. The critical density $n_c = 0.161 \times 10^{10} \text{ cm}^{-2}$.

which are completely realistic, clearly bring out the fact that, when the underlying disorder has a basic 2-impurity model structure (one type of impurity with $k_F d \gg 1$ and the other type with $k_F d \ll 1$), mobility and quality of individual samples may very well be completely independent quantities.

We believe that the results of Fig. 5 completely qualitatively (perhaps even quantitatively) explain the recent “puzzling” finding¹² that the fragile 5/2 fractional quantum Hall effect (FQHE), which is traditionally only studied in the highest quality samples with $\mu > 10^7 \text{ cm}^2/\text{Vs}$, can actually be observed in much lower mobility samples with $\mu \sim 10^6 \text{ cm}^2/\text{Vs}$ since, under suitable circumstances (as in the results of Fig. 5), it is possible for samples with orders of magnitude different mobilities (i.e. values of τ_t) to have more or less the same “quality” (i.e., the same value of τ_q).

In Figs. 6 – 9, we make the above issue very clear by showing realistic transport calculation results (for both τ_t and τ_q) in various situations within the 2-impurity model. In each case, the high carrier density mobility is determined by the background impurity scattering whereas the quality, i.e., the quantum lifetime τ_q (or equivalently the single-particle level broadening $\Gamma \sim \tau_q^{-1}$) is determined by remote impurity scattering, creating a clear dichotomy where mobility and quality are disconnected and the high-density mobility by itself does not provide a unique characterization of the sample quality.

To make the physical implication of the mobility/quality dichotomy very explicit, we have shown in each figure the carrier density where the Ioffe-Regel criterion for strong localization, $\Gamma = E_F$, is satisfied in each of these samples.¹³ (We mention that Figs. 6 – 9 should be thought of as representing five distinct 2D samples with fixed bare disorder each as described in each figure caption, but with variable 2D carrier density, as for example, can be implemented experimentally using an external back gate.) This $\Gamma = E_F$ Ioffe-Regel point should be thought of as the critical density below (above) which the system behaves insulating (metallic) as has recently been discussed by us in details elsewhere¹³. Such

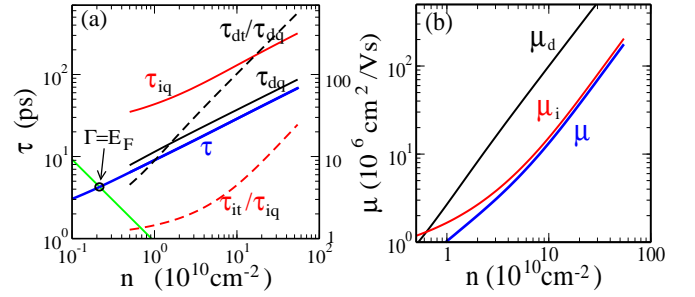


FIG. 8. (a) The same as Fig. 6 with following parameters: $n_i(d=0) = 3 \times 10^8 \text{ cm}^{-2}$, $n_d(d=500\text{\AA}) = 5 \times 10^9 \text{ cm}^{-2}$, and quantum well width $a = 300 \text{\AA}$. At a density $n = 10^{11} \text{ cm}^{-2}$ we have $\mu = 13.3 \times 10^6 \text{ cm}^2/\text{Vs}$ and $\tau_q = 28.8 \text{ ps}$. The critical density $n_c = 0.215 \times 10^{10} \text{ cm}^{-2}$.

an apparent disorder driven effective 2D metal-insulator transition (2D MIT) has been extensively studied in the literature¹⁴, and is usually discussed in terms of the maximum mobility of the sample at high carrier density.

One can think of the Ioffe-Regel criterion induced critical density n_c to be an approximate quantitative measure of the “sample quality” with n_c decreasing (increasing) as the quality improves. The corresponding approximate measure of the sample mobility has traditionally been the so-called “maximum mobility (μ_m)”, or equivalently for high-mobility GaAs-based modulation-doped structures, the measured mobility at the highest possible carrier density (since for modulation-doped high-mobility structures, as can be seen in Figs. 6 – 9 and as has been extensively experimentally observed over the last 20 years, the sample mobility decreases with decreasing carrier density and the typically quoted sample mobility is always the one measured at the highest carrier density). The naïve expectation is that higher (lower) the maximum mobility, lower (higher) should be the critical density for 2D MIT since the sample quality should improve with sample mobility.

Using μ_m to be the mobility at $n = 10^{11} \text{ cm}^{-2}$ in Figs. 6 – 9, we conclude that the sample quality, if it is indeed determined entirely by the maximum mobility (or the mobility at a very high carrier density), should decrease monotonically as we go from the sample of Fig. 6 ($\mu_m = 35.7 \times 10^6 \text{ cm}^2/\text{Vs}$) to that of Fig. 9 ($\mu_m = 11.7 \times 10^6 \text{ cm}^2/\text{Vs}$). We list below in Table I the calculated critical density for each sample in Figs. 6 – 9 noting also the mobility μ_m at $n = 10^{11} \text{ cm}^{-2}$.

Figure	$\mu_m (10^6 \text{ cm}^2/\text{Vs})$	$n_c (10^{10} \text{ cm}^{-2})$
Fig. 6	35.7	0.24
Fig. 7	14.1	0.16
Fig. 8	13.3	0.22
Fig. 9	11.7	0.33

TABLE I. μ_m is the mobility calculated at $n = 10^{11} \text{ cm}^{-2}$ and n_c represents the critical density calculated from $\Gamma = E_F$.

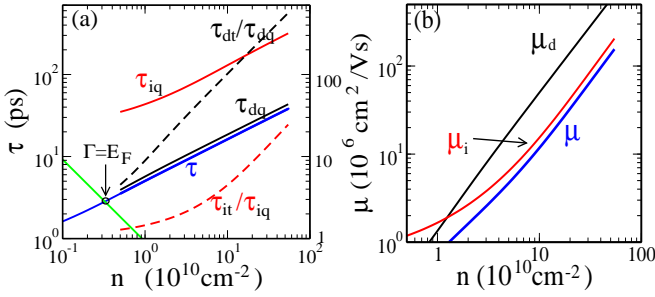


FIG. 9. (a) The same as Fig. 6 with following parameters: $n_i(d=0) = 3 \times 10^8 \text{ cm}^{-2}$, $n_d(d=500\text{\AA}) = 10^{10} \text{ cm}^{-2}$, and quantum well width $a = 300 \text{ \AA}$. At a density $n = 10^{11} \text{ cm}^{-2}$ we have $\mu = 11.7 \times 10^6 \text{ cm}^2/\text{Vs}$ and $\tau_q = 16.2 \text{ ps}$. The critical density $n_c = 0.326 \times 10^{10} \text{ cm}^{-2}$.

We conclude from Table I that there is simply no one-to-one relationship between mobility and quality in these numerical transport results based on the 2-impurity model. For example, although the “lowest mobility” sample (Fig. 9, $\mu_m = 11.7 \times 10^6 \text{ cm}^2/\text{Vs}$) does indeed have the “lowest quality” as reflected in the highest value of n_c ($\sim 0.33 \times 10^{10} \text{ cm}^{-2}$), the highest mobility sample (with almost three times the mobility of all the other samples) has the second highest value of n_c ($\sim 0.24 \times 10^{11} \text{ cm}^{-2}$) instead of having the lowest n_c as it would if quality is determined exclusively by mobility. The other two intermediate mobility samples with mobilities $14.1 \times 10^6 \text{ cm}^2/\text{Vs}$ and $13.3 \times 10^6 \text{ cm}^2/\text{Vs}$ also have their n_c values “reversed” ($0.16 \times 10^{10} \text{ cm}^{-2}$ and $0.22 \times 10^{10} \text{ cm}^{-2}$, respectively) compared with what they should be if the mobility really determined quality. We note that the samples of Figs. 6 and 8 have almost identical quality (i.e., essentially the same values of n_c) although the sample of Fig. 6 has almost three times the high-density mobility as that of Fig. 8!

We do mention that the values of n_c ($\sim 2 \times 10^9 \text{ cm}^{-2}$) we obtain in our Figs. 6 – 9 are consistent with the observed 2D MIT critical density in ultra-high-mobility 2D GaAs structures where $n_c \sim 2 \times 10^9 \text{ cm}^{-2}$ has been reported for $\mu_m \sim 10^7 \text{ cm}^2/\text{Vs}$ for $n \lesssim 10^{11} \text{ cm}^{-2}$.¹⁵

The last set of numerical results we show for the 2-impurity model is based on HIGFET (heterojunction-insulator-gated-field-effect-transistor) structures (in contrast to MODFET structures or modulation-doped-field-effect-transistors, which we have discussed so far in this paper) and is motivated by the recent experimental work by Pan and his collaborators on the effect of disorder on the observation, existence, and stability of the 5/2 FQHE in high-mobility GaAs-AlGaAs HIGFET structures¹⁶. This work of Pan *et al.* is closely related to similar work by Gamez and Muraki¹² and by Samkharadze *et al.*¹⁷ who also studied disorder effects on the stability of the 5/2 FQHE in modulation doped GaAs-AlGaAs 2D systems. All three of these experimental studies conclude, using different phenomenology and methodology, that the quality of the observed 5/2 FQHE in 2D systems

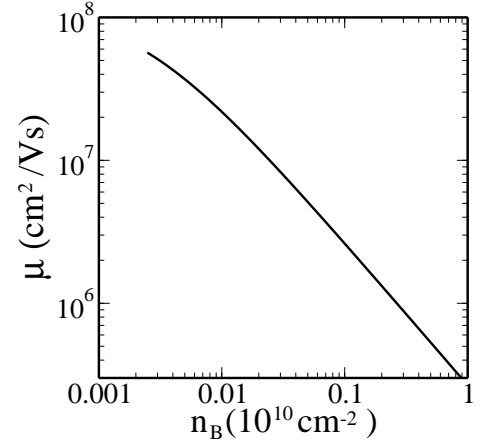


FIG. 10. (a) The calculated mobility as a function of background impurity density n_B in a GaAs HIGFET structure. Here $n_R = 10^{13} \text{ cm}^{-2}$, $d_R = 630 \text{ nm}$, and the carrier density $n = 4.7 \times 10^{11} \text{ cm}^{-2}$ are used.

is not directly connected in an one-to-one manner with the sample mobility, and it is possible to find robust 5/2 FQHE in samples with mobility in the $\gtrsim 10^6 \text{ cm}^2/\text{Vs}$ range whereas much of the earlier work^{18,19} had to use ultra-high mobility ($> 10^7 \text{ cm}^2/\text{Vs}$) for the observation of stable 5/2 FQHE. This observation by these three experimental groups of the mobility/quality dichotomy is very similar to the theory being developed in the current work with the only difference being that our work specifically focuses on the quality being associated with the single-particle quantum scattering rate τ_q^{-1} or the collisional level-broadening $\Gamma \sim \tau_q^{-1}$ rather than the 5/2 FQHE gap since we do not know of any quantitative microscopic theory which directly connects FQHE gap values with disorder. We comment further on this feature below in our discussion after presenting our HIGFET numerical results.

A HIGFET system is different from modulation-doped quantum well structures we considered so far in our work with the important qualitative difference being that HIGFETs are undoped (except, of course, for unintentional background charged impurities as represented by n_B , which are unavoidable in a semiconductor) with no remote modulation doping layer present in the system. Instead, the 2D carriers are induced in the GaAs surface layer at the AlGaAs-GaAs interface by a remote heavily doped gate placed very far from the GaAs-AlGaAs interface.²⁰ Thus, HIGFETs are basically the GaAs version of Si-MOSFETs (metal-oxide-semiconductor-field-effect transistors) with the insulator being the AlGaAs layer instead of SiO_2 . An additional difference between HIGFETs and modulation-doped quantum wells is that the quasi-2D carrier confinement in the HIGFET is in an asymmetric triangular potential well (similar to MOSFETs⁹) in contrast to the symmetric square well confinement in the AlGaAs-GaAs quantum well system.

Given that HIGFETs have no intentional modulation

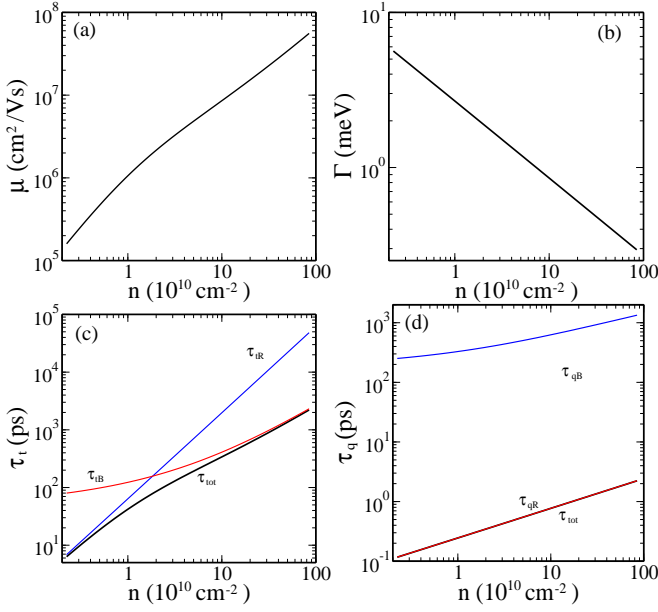


FIG. 11. The calculated (a) mobility, μ , (b) level broadening, Γ , (c) transport lifetime, τ_t and (d) quantum lifetime, τ_q , as a function of carrier density in a GaAs HIGFET structure with parameters $n_R = 10^{13} \text{ cm}^{-2}$, $d_R = 630 \text{ nm}$, and $n_B = 1.69 \times 10^8 \text{ cm}^{-2}$.

doping, it may appear that the 2-impurity model is simply inapplicable here since the background unintentional charged impurities seem to be the only possible type of Coulomb disorder in the system so that the system should belong to a 1-impurity disorder description (i.e., just the unintentional background random charged impurities). This is, however, incorrect because the presence of the far-away gate, which induces the 2D carriers in the HIGFET, introduces remote charged disorder (albeit at a very large value of d) arising from the gate charges which must be present due to the requirement of charge neutrality. We, therefore, use exactly the same minimal 2-impurity model for the HIGFETs that we have used for the modulation doped systems assuming n_R to be the charged impurity density on the far away gate at a very large distance d_R away from the induced 2D electron layer on the GaAs side of the GaAs-AlGaAs interface. (Later in this section we will present results for a realistic 3-impurity model in order to provide a quantitative comparison with the HIGFET data of Pan *et al.*¹⁶.)

In Figs. 10 – 12 we show our full numerical results of a n-GaAs HIGFET structure using the 2-impurity model. The specific HIGFET structure used for our numerical calculations is motivated by the sample used in Ref. [16], but we do not attempt any quantitative comparison with the experimental transport results, which necessitates a 3-impurity model to be described later. At this stage, i.e., for Figs. 10 – 12, our goal is to establish the mobility/quality dichotomy for HIGFET 2D systems based on our minimal 2-impurity model.

In Fig. 10 we show the calculated mobility as a function

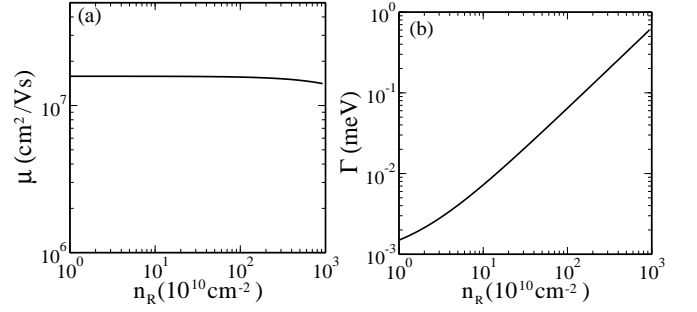


FIG. 12. The calculated (a) mobility and (b) level broadening as a function of remote impurity density in a GaAs HIGFET structure with parameters $n = 1.8 \times 10^{11} \text{ cm}^{-2}$, $d_R = 630 \text{ nm}$, and $n_B = 1.69 \times 10^{10} \text{ cm}^{-2}$.

of n_B , the background impurity density and in Fig. 11 we show the mobility (μ), the level broadening (Γ), the transport lifetime (τ_t), and the quantum lifetime (τ_q) as a function of the 2D carrier density n in a GaAs HIGFET structure using the 2-impurity model with $n_R = 10^{13} \text{ cm}^{-2}$; $d_R = 630 \text{ nm}$; $n_B = 1.69 \times 10^8 \text{ cm}^{-2}$. We note that $\mu = ne\tau_t$ and $\Gamma = \hbar/2\tau_q$ are simple measures of mobility and quality which are directly linearly connected to τ_t and τ_q^{-1} . Our choice of $d_R = 630 \text{ nm}$ is specifically aimed at the sample of Ref. [16] where the gate is located 630 nm away from the GaAs-AlGaAs interface. Our choice of $n_R = 10^{13} \text{ cm}^{-2}$ and $n_B = 1.69 \times 10^8 \text{ cm}^{-2}$ is arbitrary at this stage (and the precise choice here is irrelevant with respect to our qualitative conclusions) except that this combination of a large (small) n_R (n_B) is the appropriate physical situation in high-quality HIGFETs. Our choice of n_R , n_B , and d_R (which we get from the actual experimental system) gives the correct 2D “maximum” mobility of $\mu = 14 \times 10^6 \text{ cm}^2/\text{Vs}$ at a 2D carrier density of $n = 4.7 \times 10^{11} \text{ cm}^{-2}$ consistent with the experimental sample in Ref. [16] as shown in Fig. 10.

The calculated mobility in Fig. 10 decreases monotonically with increasing n_B , and we choose $n_B = 1.69 \times 10^8 \text{ cm}^{-2}$ to get the correct maximum mobility of $14 \times 10^6 \text{ cm}^2/\text{Vs}$ reported in Ref. [16] with the corresponding value of the level broadening being 0.638 meV at the same density. We emphasize that the level broadening (or equivalently, τ_q) here is determined entirely by the remote scattering from the gate in spite of the gate being an almost macroscopic distance ($\sim 0.6 \mu\text{m}$) away from the 2D electrons – changing n_B by even a factor of 100 does not change the value of Γ or τ_q^{-1} (but does change mobility μ or τ_t^{-1} by a factor of 100) whereas the mobility is determined entirely by the background scattering (and therefore changing n_R does not affect the mobility).

In Fig. 11 we show the calculated μ , Γ , τ_t , and τ_q (remembering $\mu = ne\tau_t$ and $\Gamma = \hbar/2\tau_q$) as a function of 2D carrier density n for fixed n_R , d_R , n_B as shown. These results clearly show the mobility/quality dichotomy operational within the 2-impurity model in this particular HIGFET structure (for the chosen realis-

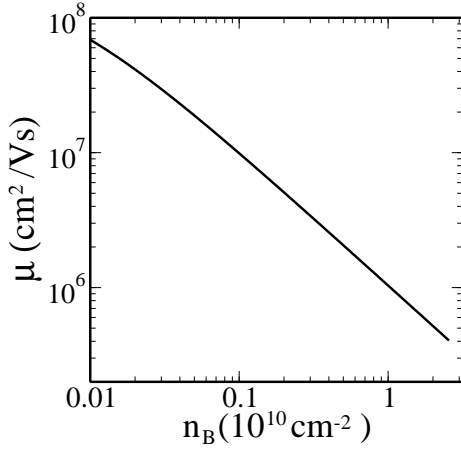


FIG. 13. (a) The calculated mobility as a function of background impurity density n_B in a GaAs MODFET structure with a well width $a = 300\text{\AA}$. Here $n = n_R = 1.8 \times 10^{11} \text{ cm}^{-2}$ and $d_R = 2000 \text{ nm}$ are used.

tic disorder parameters n_R , d_R , n_B). For $n \gtrsim 2 \times 10^{10} \text{ cm}^{-2}$, the mobility is determined essentially by the background impurity scattering (i.e. n_B) whereas the level broadening or the quantum scattering rate is determined entirely by the remote scattering for the entire density range ($10^9 \text{ cm}^{-2} < n < 10^{12} \text{ cm}^{-2}$) shown in Fig. 11. At low carrier density ($n \lesssim 10^{10} \text{ cm}^{-2}$) $k_F d_R$ ($\lesssim 10$) is no longer very large, and given the rather large value of n_R ($= 10^{13} \text{ cm}^{-2}$) corresponding to the remote gate charges, the scattering by n_R starts affecting the mobility. But, the high density mobility (determined by n_B) and the quality at all density (determined by n_R) are still completely independent quantities, and therefore it is possible for the quality (e.g., the FQHE gap at high density) to be completely independent of the mobility as found experimentally in Ref. [16].

This is demonstrated explicitly in Fig. 12 where we show that the variation in the mobility is essentially non-existent for four orders of magnitude changes in n_R whereas Γ changes essentially by four orders of magnitude. Similarly, Fig. 11 indicates that (since $\tau_{qB}^{-1} \propto \Gamma \propto n_B$), a 2 orders of magnitude change in n_B will hardly change Γ , but μ will change by 2 orders of magnitude (due to a 2-orders of magnitude change in n_B) at high carrier density.

We believe that our Figs. 10 – 12 provide a complete explanation for the puzzling observation in Ref. [16] where a drop in the mobility of the sample at high carrier density hardly affected its quality as reflected in the measured 5/2 FQHE energy gap. This is because the high carrier density mobility is determined by background impurity density n_B which does not affect the quality at all whereas the quality is affected by remote scattering which does not much affect the mobility at high carrier density. For the sake of completeness, and to make connection with the interesting recent works of Refs. [12 and 17], who also independently conclude in agreement with Pan

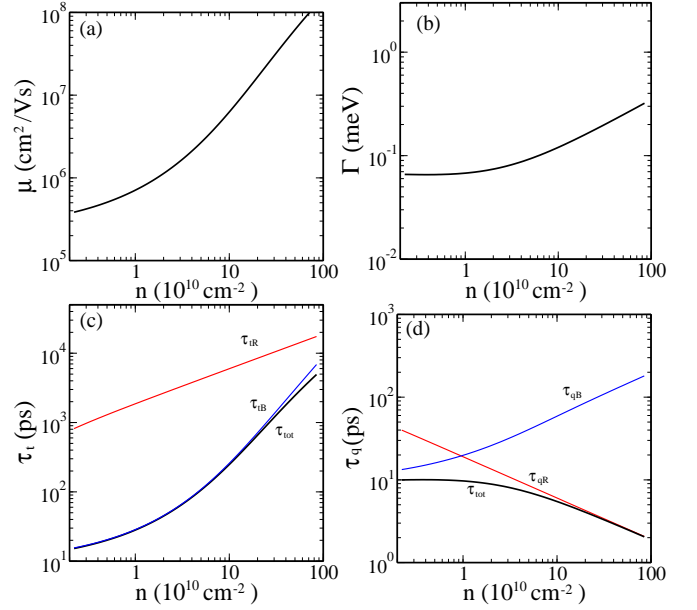


FIG. 14. The calculated (a) mobility, μ , (b) level broadening, Γ , (c) transport lifetime, τ_t and (d) quantum lifetime, τ_q , as a function of carrier density in a GaAs MODFET structure with a well width $a = 300 \text{ \AA}$. Here the parameters $n_R = n$, $d_R = 200 \text{ nm}$, and $n_B = 6.8 \times 10^8 \text{ cm}^{-2}$ are used.

*et al.*¹⁶ that very high mobility ($> 10^7 \text{ cm}^2/\text{Vs}$) is not necessarily required for the experimental observation of a robust 5/2 FQHE in standard modulation-doped GaAs quantum wells (in contrast to Pan's usage of undoped HIGFETs), we show in Figs. 13 – 15 (which correspond to the HIGFET results shown in Figs. 10 – 12 respectively) our calculated transport results for a modulation-doped quantum well structure with a high-density mobility identical (i.e., $14 \times 10^6 \text{ cm}^2/\text{Vs}$) to the HIGFET structure considered in Figs. 10 – 12.

The main differences between the 2D systems for Figs. 10 – 12 (HIGFET) and Figs. 13 – 15 (MODFET) are the following: (1) the HIGFET has a triangular quasi-2D confinement potential (determined self-consistently by the carrier density) and the MODFET has a square-well confinement imposed by the MBE-grown AlGaAs-GaAs-AlGaAs structure with a given confinement width ($a = 30 \text{ nm}$ in Figs. 13 – 15); (2) the 2D carriers are induced by a very far away gate in the HIGFET whereas it is induced by the remote dopants (we choose $n_R = n$ in Figs. 13 – 15) in the modulation doping layer (we choose $d_R = 200 \text{ nm}$ in Figs. 13 – 15); (3) the specific necessary values of n_B are somewhat different in the two systems in order to produce the same high-density maximum mobility. The quantitative differences described in items (1) – (3) above are sufficient to produce substantial differences between the numerical results in the HIGFET and the MODFET system as can easily be seen by comparing the results of Figs. 10 – 12 with those of Figs. 13 – 15, respectively, although we ensured that both have exactly the same high-density mobility ($\mu_m = 1.4 \times 10^7 \text{ cm}^2/\text{Vs}$).

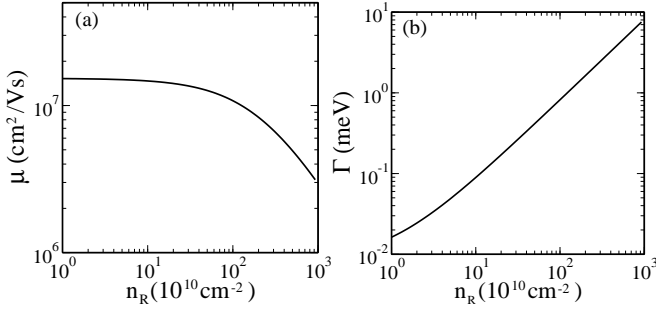


FIG. 15. The calculated (a) mobility and (b) level broadening as a function of remote impurity density in a GaAs MOD-FET structure with a well width $a = 300 \text{ \AA}$. The parameters $n = 1.8 \times 10^{11} \text{ cm}^{-2}$, $d_R = 200 \text{ nm}$, and $n_B = 6.8 \times 10^{10} \text{ cm}^{-2}$ are used.

However, qualitatively the two sets of results shown in Figs. 10 – 12 and 13 – 15 are similar in that the mobility (quality) at high carrier density ($> 10^{10} \text{ cm}^{-2}$) is invariably determined by the background (remote) scattering respectively, leading to the possibility that a substantial change in mobility (quality) by changing n_B (n_R) respectively may not at all affect quality (mobility), and thus it is possible at high carrier density for a system to have a modest mobility ($\sim 10^6 \text{ cm}^2/\text{Vs}$) by having a large n_B with little adverse effect on quality (i.e., Γ). Thus, the experimental observations in Refs. [12, 16, and 17] are all consistent with our theoretical results.

Finally, we show in Figs. 16 and 17 the numerical transport results for the HIGFET structure (of Figs. 10 – 12) using a more realistic 3-impurity model going beyond the 2-impurity model mostly used in our current work. The 3-impurity model is necessary for obtaining agreement between experiment¹⁶ and theory since experimentally the measured mobility, $\mu(n)$, as a function of carrier density manifests non-monotonicity with a maximum in the mobility around $n \sim 2 \times 10^{11} \text{ cm}^{-2}$. Such a non-monotonicity, where μ increases (decreases) with increasing n at low (high) carrier density, is common in Si-MOSFETs⁹, and is known to arise from short-range interface scattering which becomes stronger with increasing carrier density as the self-consistent confinement of the 2D carriers becomes stronger and narrower pushing the electrons close to the interface and thus increasing the short-range interface roughness scattering as well as the alloy disorder scattering in AlGaAs as the confining wave function tail of the 2D electrons on the GaAs side pushes into the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ side of the barrier. We include this realistic short-range scattering effect, which becomes important at higher carrier density leading to a decrease of the mobility at high density (as can be seen in Fig. 16(a)). Importantly, however, this higher-density suppression of mobility (by a factor of 3 in Fig. 16(a) consistent with the observation of Pan *et al.*¹⁶) has absolutely no effect on the quality (see Fig. 16(b)) with the level broadening Γ decreasing monotonically with increasing carrier density (since Γ is determined essentially entirely by the

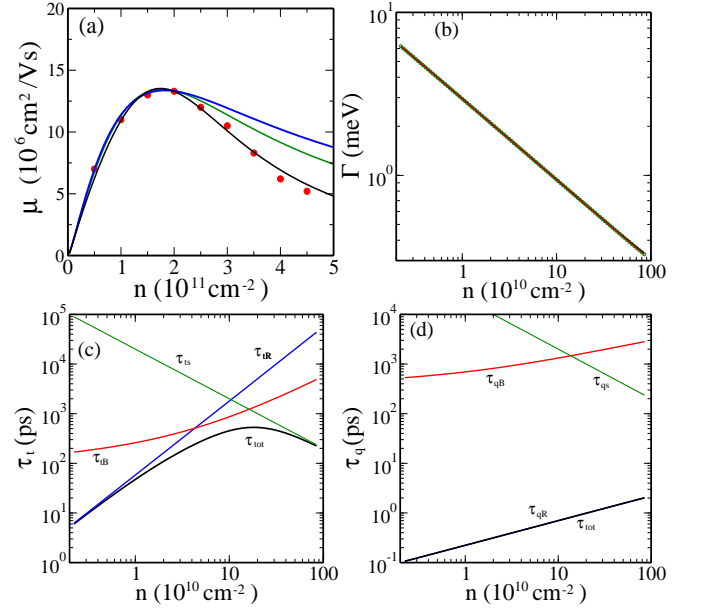


FIG. 16. The calculated (a) mobility and (b) level broadening with long range remote impurity at d_R , short range impurities at the interface, and background short range impurities. We assume that the density dependence of scattering time with interface short range impurities is $\tau_{qs}^{-1} \propto n^\alpha$. In (a) the blue line is calculated with $n_B = 0.8 \times 10^8 \text{ cm}^{-2}$ and $\tau_{qs}^{-1} = 5 \times 10^7 n$. The green line is calculated with $n_B = 0.9 \times 10^8 \text{ cm}^{-2}$ and $\tau_{qs}^{-1} = 1.85 \times 10^7 n^{1.3}$. The black line is calculated with $n_B = 1.08 \times 10^8 \text{ cm}^{-2}$ and $\tau_{qs}^{-1} = 0.19 \times 10^7 n^2$. The red dots are experimental data from Pan *et al.*¹⁶ In (c) and (d) we show the total scattering times as well as the individual scattering time corresponding to the each scattering source. Here $n_B = 0.8 \times 10^8 \text{ cm}^{-2}$ and $\tau_{qs}^{-1} = 5 \times 10^7 n$ are used.

remote scattering – see Fig. 16(d)). Thus, we see an apparent paradoxical situation (compare Figs. 16(a) and (b)) where the mobility decreases at higher carrier density, but the quality keeps on improving with increasing carrier density! This is precisely the phenomenon observed by Pan *et al.*¹⁶ who found that, although the mobility itself decreased in their sample by a factor of 3 at higher density, the sample quality, as measured by the 5/2 FQHE gap, improved with increasing density precisely as we predict in our work. In Fig. 17 we show that the 3-impurity model, except for allowing the mobility to decrease at high carrier density due to the increasing dominance of short-range scattering (thus bringing experiment and theory into agreement at high density in contrast to the 2-impurity results), has no effect on the basic quality/mobility dichotomy being discussed in this work – for example, Fig. 17 shows that while quality decreases (i.e., Γ increases) monotonically with increasing remote scattering, nothing basically happens to the mobility!

Before concluding this section, we provide a critical and quantitative theoretical discussion of two distinct experiments (one from 1993²¹ and the other from 2011¹⁷),

separated by almost 20 years in time, involving high-mobility 2D semiconductor structures in the context of the mobility versus quality question being addressed in the current work. Our reason for focusing on these two papers is because both report τ_t and τ_q for the samples used in these experimental studies, thus enabling us to apply our theoretical analyses quantitatively to these samples.

In ref. [21], two GaAs-AlGaAs heterojunctions were used (samples A and B) with the following characteristics²²: Sample A: $n = 1.1 \times 10^{11} \text{ cm}^{-2}$; $\mu = 6.8 \times 10^6 \text{ cm}^2/\text{Vs}$; $\tau_t = 270 \text{ ps}$; $\tau_q = 9 \text{ ps}$, and Sample B: $n = 2.3 \times 10^{11} \text{ cm}^{-2}$; $\mu = 12 \times 10^6 \text{ cm}^2/\text{Vs}$; $\tau_t = 480 \text{ ps}$; $\tau_q = 4.5 \text{ ps}$. Both samples A and B have the same setback distance of $d_R = 80 \text{ nm}$ for the remote dopants, but we should consider $d_A > d_B \gtrsim 80 \text{ nm}$ since sample A has a lower carrier density and therefore the quasi-2D layer thickness for sample A must be slightly higher since the self-consistent confinement potential must be weaker in A than in B due to its lower density. We note that sample A and B indeed manifest the mobility/quality dichotomy in that A (B) has higher (lower) quality (i.e. τ_q), but lower (higher) mobility!

We start by assuming the absence of any background impurity scattering ($n_B = 0$), then the asymptotic formula for $k_F d_R \gg 1$ applies to both samples, giving, $\tau_t \sim (k_F d_R)^3 / n_R$; $\tau_q \sim (k_F d_R) / n_R$. Making the usual assumption $n_R = n$, since no independent information is available for n_R , we conclude that the theory predicts $\tau_t^B / \tau_t^A = \sqrt{n_B / n_A} (d_B / d_A)^3 \approx 1.4$ assuming $d_B \approx d_A$, and $\tau_q^B / \tau_q^A = \sqrt{n_A / n_B} d_B / d_A \approx 0.7$ assuming $d_B \approx d_A$. Experimentally, A and B samples satisfy: $\tau_t^B / \tau_t^A \approx 1.8$; $\tau_q^B / \tau_q^A = 0.5$. Thus, just the consideration of only remote dopant scattering which must always be present in all modulation-doped samples already gives semi-quantitative agreement between theory and experiment including an explanation of the apparent paradoxical finding that the sample B with higher mobility has a lower quality! The key here is that the higher density of sample B leads to a higher mobility, but also leads to a higher values of τ_q^{-1} (and hence lower quality) by virtue of higher carrier density necessitating a higher value of n_R leading to a lower value of τ_q [see, for example, Fig. 1(c) where increasing $n = n_R$ leads to increasing (decreasing) τ_t (τ_q)].

We can actually get essentially precise agreement between theory and experiment for the dichotomy in samples A and B of Ref. [21], with τ_t higher (lower) in sample B (A) and τ_q higher (lower) in sample A (B) by incorporating the fact that a higher (by a factor of 2) carrier density in sample B compared with sample A makes $d_A > d_B$ due to self-consistent confinement effect in heterostructures and hence the theoretical ratios of τ^A and τ^B change from the values given above to $\tau_t^B / \tau_t^A < 1.4$ and $\tau_q^B / \tau_q^A \approx 0.5$ (i.e., < 0.7). This means that while the quality ratio τ_q^B / τ_q^A of samples A and B can be understood quantitatively on the basis of remote scattering (which determines the quality almost exclusively in

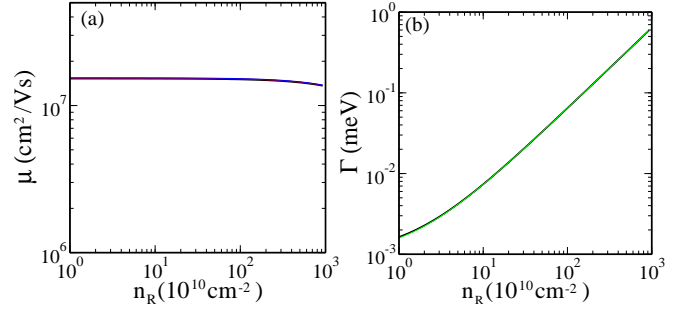


FIG. 17. The calculated (a) mobility and (b) level broadening as a function of the remote impurity density. The same parameters as Fig. 16(a) are used.

high-mobility modulation-doped structures), the mobility ratio τ_t^B / τ_t^A is not determined exclusively by remote dopant scattering. Inclusion of somewhat stronger background disorder scattering in sample A compared with sample B immediately gives $\tau_t^B / \tau_t^A = 1.8$ in agreement with experiment. Thus, we see as asserted by us theoretically, quality and mobility are mainly controlled by distinct scattering mechanisms (quality by remote scattering and mobility by background scattering) in the data of ref. [21] providing an explicit example of the mobility/quality dichotomy as far back as in 1993 when this dichotomy was not discussed at all in the literature.

Considering now the samples used in ref. [17], there are again two distinct modulation-doped quantum well samples with the following sample specifications: Sample A: $a = 56 \text{ nm}$; $d_R = 320 \text{ nm}$; $n = 8.3 \times 10^{10} \text{ cm}^{-2}$; $\mu = 12 \times 10^6 \text{ cm}^2/\text{Vs}$; $\Gamma = 0.24 \text{ K}$, and Sample B: $a = 30 \text{ nm}$; $d_R = 78 \text{ nm}$; $n = 2.78 \times 10^{11} \text{ cm}^{-2}$; $\mu = 11 \times 10^6 \text{ cm}^2/\text{Vs}$; $\Gamma = 2.04 \text{ K}$. Thus, in this case¹⁷, although the two samples have almost identical mobilities, the lower-density sample A has almost 8 times higher quality with $\Gamma_B / \Gamma_A = \tau_q^A / \tau_q^B \approx 8$. We note that the lower quality sample has three times the carrier density, and going back to our Figs. 1 – 4, we see that a higher carrier density n ($= n_R$) always leads to higher mobility and lower quality since the quality (i.e., τ_q) is determined mostly by long-range remote scattering whereas the mobility is determined by a combination of both remote and background scattering with the background scattering often dominating the mobility. The fact that $d_R^A \gg d_R^B$ considerably improves the quality of sample A with respect to sample B without much affecting the mobility since the quality (mobility) is limited by remote (background) scattering.

Using the asymptotic formula (for $k_F d \gg 1$), $\tau_q \propto k_F d / n_R$ and $n_R \approx n$, we conclude for the comparative quality of the two samples: $\tau_q^A / \tau_q^B = \Gamma_B / \Gamma_A \approx \sqrt{n_B / n_A} d_A / d_B \approx 8$ where we use $d_A = 348 \text{ nm}$ and $d_B = 93 \text{ nm}$ by taking into account their differences in both the set back distances and the well thickness. Experimentally, $\Gamma_B / \Gamma_A \approx 8.5$ in excellent agreement with the theoretical estimate. The fact that the mobilities of

A and B are similar is easily explained by their similarity with respect to background disorder with sample B having somewhat larger value of unintentional background impurity density than sample A. Thus, our mobility/quality theoretical dichotomy is in perfect accord with the data of ref. [17].

We now conclude this section by mentioning that we have used the quantum lifetime (or the single particle scattering time) τ_q (or equivalently $\Gamma \propto \tau_q^{-1}$) as a measure of the quality because it is well-defined and theoretically calculable. Experimentally, the quality can be defined in a number of alternative ways as, for example, done in the recent experiments^{12,16,17} where the 5/2 FQHE gap is used as a measure of the quality. There is no precise microscopic theory for calculating disorder effects on the FQHE gap, but there are strong indications^{17,19,23} that the FQHE gap Δ_Γ in the presence of finite disorder scales approximately as

$$\Delta_\Gamma \approx \Delta_0 - \Gamma, \quad (36)$$

where Γ is indeed the quantum level broadening we use in our current work as the measure of quality and Δ_0 is the FQHE gap in the absence of any disorder. If this is even approximately true (as it seems to be on empirical grounds), then our current theoretical work shows complete consistency with the recent experimental results concerning the dichotomy between mobility and FQHE gap values in the presence of disorder. In this context, it may be worthwhile to emphasize an often overlooked fact: the mobility itself (i.e., τ_t^{-1} and *not* τ_q^{-1}) can be converted into an energy scale by writing (for GaAs)

$$\Gamma_\mu = \frac{\hbar}{2\tau_t} \approx (10^{-4}/\tilde{\mu}) \text{ meV} \approx (.01/\tilde{\mu}) \text{ K}, \quad (37)$$

where $\tilde{\mu} = \mu/(10^7 \text{ cm}^2/\text{Vs})$. Thus, a mobility of $10^7 \text{ cm}^2/\text{Vs}$ corresponds only to a broadening of 10 mK which is miniscule compared with the theoretically calculated^{23,24} 5/2 FQHE gap of 2 – 3 K! Even a mobility of $10^6 \text{ cm}^2/\text{Vs}$ corresponds to a mobility broadening of only 100 mK, which is much less than the expected 5/2 FQHE gap. Thus, the quality of the 5/2 FQHE cannot possibly be determined directly by the mobility value (unless the mobility is well below $10^6 \text{ cm}^2/\text{Vs}$) and there must be some other factor controlling the quality, which we take to be the quantum level broadening in this work. It must be emphasized here that the mobility/quality dichotomy obviously arises from the underlying disorder in high-mobility semiconductor structures being long-ranged. If both mobility and quality are dominated by short-range disorder, then $\tau_q \approx \tau_t$, and a mobility of $10^6 \text{ cm}^2/\text{Vs}$ with $\Gamma \approx 100 \text{ mK}$ will be a high-quality sample!

In concluding this section, we should mention that the very first experimental work we know of where the mobility/quality dichotomy was demonstrated and noted explicitly in the context of FQHE physics is a paper by Sajoto et al.²⁵ from the Princeton group which appeared in print an astonishing 24 years ago! In this work, (see

the “Note added in proof” in Ref. [25]), it was specifically stated that the samples used by Sajoto et al. manifested as strong FQHE states as those observed in other samples from other groups with roughly 5 – 10 times the mobility of the Sajoto et al. samples, thus providing a clear and remarkable early example of the mobility/quality dichotomy much discussed during the last couple of years in the experimental 2D literature. We note that the samples used by Sajoto et al.²⁵ had unusually large set-back distances ($d_R \sim 270 \text{ nm}$), leading to rather small values of τ_q^{-1} and Γ corresponding to our theory although the mobility itself, being limited by background impurity scattering (i.e. by n_B), was rather poor ($\sim 10^6 \text{ cm}^2/\text{Vs}$). We believe that the reason the samples of Sajoto et al. had such high quality in spite of having rather modest mobility is the mobility/quality dichotomy studied in our work where the mobility determined by background scattering is disconnected from the quality determined by the remote dopant scattering.

IV. SUMMARY AND CONCLUSION

In summary, we have theoretically discussed the important issue of mobility versus quality in high-mobility 2D semiconductor systems such as modulation-doped GaAs-AlGaAs quantum wells and GaAs undoped HIGFET structures. We have established, both analytically (section II) and numerically (section III), that modulation-doped (or gated) 2D systems should generically manifest a mobility/quality dichotomy, as often observed experimentally, due to the simple fact that mobility and quality are often determined by different underlying disorder mechanisms in 2D semiconductor structures – in particular, we show definitively that in many typical situations, the mobility (quality) is controlled by near (far) quenched charged impurities, particularly at higher carrier density and higher mobility samples. We show that often the 2D mobility (or equivalently, the 2D transport scattering time) is controlled by the unintentional background charged impurities in the 2D layer whereas the quality, which we have parameterized throughout our work by the quantum single-particle scattering time (or equivalently, the quantum level broadening), is controlled by the remote charged impurities in the modulation doping layer whose presence is necessary for inducing carriers in the 2D layer. Somewhat surprisingly, we show that the same mobility/quality dichotomy could actually apply to undoped HIGFET structures where the charges on the far-away gate play the role of remote scattering mechanism. Quite unexpectedly, we show that a very far away gate (located almost 10^{-4} cm away from the 2D layer) can still completely dominate the quantum level broadening, while at the same time having no effect on the mobility. We develop a minimal 2-impurity model (near and far or background and remote) which is sufficient to explain all the observed experimental features of the mobility/quality dichotomy. The key physical point here is

that the dimensionless parameter ‘ $k_F d$ ’, where $k_F \propto \sqrt{n}$ is the Fermi wave number of the 2D electron system and ‘ d ’ is the distance of the relevant charged impurities from the 2D system completely controls the mobility/quality dichotomy. Impurities with $k_F d \gg 1$ ($\ll 1$) could totally dominate quality (mobility) without affecting the other property at all. We give several examples of situations where identical or very similar sample mobilities at high carrier density could lead to very different sample qualities (i.e., quantum level broadening differing by large factors) and vice versa. The mobility/quality dichotomy in our minimal 2-impurity model arises from the exponential suppression of the large angle scattering by remote charged impurities which leads to the interesting situation that remote scattering contributes little to the resistivity, but a lot to the level broadening through the accumulation of substantial small angle scattering. We emphasize that the mobility/quality dichotomy arises entirely from the long-range nature of the underlying disorder, and would disappear completely if the dominant disorder in the system is short-ranged.

It is important to realize that τ_t (mobility) and τ_q (quality) both depend not only on the disorder strength, but also on the carrier density, i.e., $\tau_{t,q} \equiv \tau_{t,q}(n, n_R, d_R, n_B)$. Thus, even within the 2-impurity model (parameterized by disorder parameters n_R, d_R, n_B), $\tau_{t,q}$ are both functions of carrier density. For very low carrier density, the dimensionless parameter $k_F d$ may be small for all relevant impurities in the system, and eventually our 2-impurity model will then fail since at such a low carrier density, all impurities are essentially near impurities with the distinction between R-impurities and B-impurities being merely a semantic distinction with no real difference. Mobility and quality at such low densities then will behave similarly. The same situation may also apply as a matter of principle at very high carrier densities (i.e., very large k_F) where all impurities may satisfy $k_F d \gg 1$ and thus act as far impurities, again leading to a breakdown of the 2-impurity model. This density dependence of the 2-impurity model with respect to mobility/quality dichotomy is, however, a non-issue for our current work since (1) typically, samples are characterized by their mobility values at some fixed high (but not too high) carrier density ($n \sim 10^{11} - 4 \times 10^{11} \text{ cm}^{-2}$), and (2) the very low and high density regimes where the 2-impurity model is no longer operational are completely out of the experimentally relevant density range of interest in high-mobility 2D semiconductor structures for the physics (e.g., FQHE) studied in this context. Assuming a high-mobility modulation doped GaAs quantum well of thickness 200 – 400 Å and a set-back distance of 600 – 2000 Å (these are typical numbers for high-mobility 2D GaAs structures), the 2-impurity model should be well-valid in a wide range of carrier density $5 \times 10^9 \text{ cm}^{-2} \lesssim n \lesssim 5 \times 10^{11} \text{ cm}^{-2}$, which is the applicable experimental regime of interest. Thus, the applicability of the 2-impurity model for considering the mobility/quality dichotomy is not a serious issue of con-

cern.

A second concern could be the validity (or not) of our theoretical approximation scheme for calculating $\tau_{t,q}$, where we have used the zero-temperature Boltzmann theory and the leading-order Born approximation for obtaining the scattering rates. The $T = 0$ approximation is excellent as long as $T \ll T_F = E_F/k_B$, which is valid in all systems of interest in this context. For high-mobility 2D semiconductor structures of interest in the current work, where the issue of the dichotomy of mobility/quality is relevant (since in low-mobility samples, typically $\tau_t \approx \tau_q$), the leading order theory (in the disorder strength) employed in our approximation scheme should, however, be excellent since the conditions $n \gg n_B$ and $n \gg n_R e^{-2k_F d_R}$ are both satisfied making Born approximation essentially an exact theory in this manifestly very weak disorder situation (consistent with the high carrier mobility under consideration). An equivalent way of asserting the validity of Born approximation in our theory is to note that the conditions $E_F \gg \Gamma$ and $k_F l \gg 1$ are always satisfied in the regime of our interest (with Γ and l being the quantum level broadening and the transport mean free path respectively). A related issue, which is theoretically somewhat untractable, is the possible effect of impurity correlation effects²⁶ on the mobility versus quality question in 2D semiconductor structures. It is straightforward to include impurity correlation effects among the dopant ions in our transport theory, but unfortunately no sample-dependent experimental information is available on impurity correlations for carrying out meaningful theoretical calculations. We have carried out some representative numerical calculations assuming model inter-impurity correlations among the remote dopants, finding that such correlations enhance both τ_t and τ_q , as expected (with τ_q being enhanced more than τ_t in general), compared with the completely random impurity configuration results presented in the current article, but our qualitative conclusions about the mobility/quality dichotomy remain unaffected since the fact that τ_t and τ_q are controlled respectively by background and remote scattering in high-mobility modulation-doped structures continues to apply in the presence of impurity correlation effects. We therefore believe that our current theory involving Born approximation assuming weak leading order disorder scattering from random uncorrelated quenched charged impurities in the environment (both near and far) is valid in the parameter regime of our interest.

Finally, we comment on the possibility of future experimental work to directly verify (or falsify) our theory. Throughout this paper, we have, of course, made extensive contact with the existing experimental results which, in fact, have motivated our current theoretical work on the mobility versus quality dichotomy. For a direct future experimental test of the theory, it will be necessary to produce a large number of high-mobility 2D semiconductor structures with different fixed carrier densities and with varying values of the remote dopant set-

back distance, and then measure the values of $\tau_{t,q}$ in a large set of samples which are all characterized by their high-density mobility. The measurement of the transport relaxation time τ_t is simple since it is directly connected to the carrier mobility μ (or conductivity σ): $\tau_t = m\sigma/ne^2 = m\mu/e$. The measurement of the single-particle relaxation time (or the quantum scattering time) τ_q is, however, not necessarily trivial although its theoretical definition is very simple. In particular, the Dingle temperature or equivalently the Dingle level broadening Γ_D obtained from the measured temperature dependence of the amplitude of the 2D SdH oscillations may not necessarily give the zero-field quantum scattering time τ_q defining the sample quality in our theoretical considerations (i.e. $\Gamma_D = \hbar/2\tau_q$ may not necessarily apply to the 2D SdH measurements) because of complications arising from the quantum Hall effect and inherent spatial density inhomogeneities (associated with MBE growth) in the 2D sample. Since our theory is explicitly a zero-magnetic field theory, it is more appropriate to obtain τ_q simply by carefully monitoring low-field magneto-resistance oscillations finding the minimum magnetic field B_0 where the oscillations disappear. The corresponding cyclotron energy $\omega_0 = eB_0/mc$ then defines the single particle level-broadening $\Gamma \sim \hbar\omega_0$, providing $\tau_q = 1/2\omega_0$. An advantage of this method of obtaining τ_q is that one is necessarily restricted to the low magnetic field regime in high-mobility systems (i.e., $E_F \gg \hbar\omega_0$), where our theory should be applicable. A much stronger advantage be applicable. A much stronger advantage of using this proposed definition (i.e., the disappearance of magneto-resistance oscillations at the lowest experimental temperature) for the experimental determination of τ_q is that this is much easier to implement in the laboratory than the full measurement of the Dingle temperature which requires accurate measurements of the temperature dependent SdH amplitude oscillations. We therefore sug-

gest low-temperature measurements of μ and ω_0 to obtain τ_t and τ_q respectively in a large number of modulation doped samples with varying n , n_R , d_R , and n_B in order to carry out a quantitative test of our theory. A large systematic data base of both τ_t and τ_q in many different samples should manifest poor correlations between these two scattering times (i.e., the mobility/quality dichotomy) provided the samples are high-mobility samples dominated by long-range charged impurity disorder. As emphasized (and as is well-known) throughout this work, if one type of disorder completely dominates both τ_t and τ_q , then they will obviously be correlated, but this should be more an exception than the rule in high-mobility modulation-doped 2D semiconductor structures, where both near (“ n_B ”) and far (“ n_R ”) impurities should, in general, play important roles in manifesting the mobility/quality dichotomy.

One important open question is whether the mobility/quality dichotomy we establish in the current work can be extended to other definitions of quality beyond our definition of quality in terms of the single-particle scattering rate or quantum level-broadening. The advantage of using the quantum scattering rate as the measure of sample quality is that this definition is generic, universal, and simple to calculate (and to measure). However, obviously, given an arbitrary disorder distribution involving long-range Coulomb-disorder, there are many possible definitions of quality involving many different moments of the effective Coulomb disorder. It will be very interesting for future work to choose alternate possible definitions of sample quality to establish whether our finding of the mobility/quality dichotomy applies to all possible definitions of sample quality.

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